The Trig Functions of the Special Angles

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The IB HL & SL AA (pure) Exam Paper 1 requires the student to know the trig functions of the special angles – the multiples of $\frac{\pi}{2}$, $\frac{\pi}{3}$, $\frac{\pi}{4}$ and $\frac{\pi}{6}$ – without a calculator.

How can you know the trig functions of these angles?

Think about the point (x, y) where the terminal side of the angle cuts the unit circle.

$\cos\theta = x$
$\sin\theta = y$

For the angles which are **multiples of $\frac{\pi}{2}$** the point is always on one of the axes, so the possible values of x and y are 0, ±1. If one is 0, the other must be ±1 and vice versa.

For the angles which are **multiples of $\frac{\pi}{4}$**, the possible values of x and y are $\pm\frac{\sqrt{2}}{2}$. Both must be $\pm\frac{\sqrt{2}}{2}$.

For the angles which are **multiples of $\frac{\pi}{6}$ and $\frac{\pi}{3}$**, the possible values of x and y are $\pm\frac{1}{2}$ and $\pm\frac{\sqrt{3}}{2}$. If one is $\pm\frac{\sqrt{3}}{2}$, the other must be $\pm\frac{1}{2}$ and vice versa.

The quadrant determines the sign.

Since x is positive in quadrants 1 and 4 so is $\cos\theta$.
Since y is positive in quadrants 1 and 2 so is $\sin\theta$.

Make a quick sketch, possibly in your head.

For multiples of $\frac{\pi}{6}$ and $\frac{\pi}{3}$, the drawing needs to be accurate enough to see whether x or y is bigger and then use that $\frac{1}{2} < \frac{\sqrt{3}}{2}$.

From the sketch you can see which one of the above possibilities applies to the given angle.