

# The Three Forms of a Quadratic Function (a Parabola)

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## General Form

$$y = ax^2 + bx + c$$

The *concavity* is determined by  $a$ .

If  $a > 0$  the parabola is concave up.

If  $a < 0$  the parabola is concave down.

The  $y$ -intercept is  $c$ .

The *axis of symmetry*, which is also the

$x$ -coordinate of the vertex, is  $x = \frac{-b}{2a}$ .

To find the  $x$ -intercepts, solve  $0 = ax^2 + bx + c$ :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**To get this form from the other forms:**

Multiply it out & collect like terms.

**Example**

$$y = 2x^2 - 12x - 14$$

$$a = 2, b = -12, c = -14$$

The  $y$ -intercept is  $(0, -14)$ .

The axis of symmetry is:

$$x = \frac{-(-12)}{(2)(2)} = 3. \text{ So } x = 3.$$

## Factored Form

$$y = a(x - \alpha)(x - \beta)$$

Gives the  $x$ -intercepts:  $x = \alpha, \beta$ .

To find the  $y$ -intercept set  $x = 0$  and evaluate.

The *concavity* is determined by  $a$ .

If  $a > 0$  the parabola is concave up.

If  $a < 0$  the parabola is concave down.

The  $x$ -coordinate of the vertex and the equation of the axis of the symmetry is the average of the

$x$ -intercepts,  $\frac{\alpha + \beta}{2}$ .

**To get this form from the other forms:** Factor it.

**Example**

$$y = 2(x + 1)(x - 7)$$

The  $x$ -intercepts are  $(-1, 0)$ ,  $(7, 0)$

The axis of symmetry is:

$$x = \frac{-1 + 7}{2}, \text{ so } x = 3.$$

The  $y$ -intercept is  $y = 2(0 + 1)(0 - 7) = -14$

## Vertex (or Standard) Form

$$y = a(x - h)^2 + k$$

Gives the vertex:  $(h, k)$ .

(Note the minus sign on  $h$ .)

The axis of symmetry is  $x = h$ .

The *concavity* is determined by  $a$ .

If  $a > 0$  the parabola is concave up.

If  $a < 0$  the parabola is concave down.

**To get this form from the other forms:**

Complete the square.

To find the  $x$ -intercepts set  $y = 0$  and solve for  $x$ .

To find the  $y$ -intercept set  $x = 0$  and evaluate.

**Example**

$$y = 2(x - 3)^2 - 32$$

The vertex is  $(3, -32)$

The axis of symmetry is:  $x = 3$

To find the  $x$ -intercept set  $0 = 2(x - 3)^2 - 32$

$$\text{So } 32 = 2(x - 3)^2 \text{ so } 16 = (x - 3)^2,$$

$$\text{so } \pm 4 = x - 3, \text{ so } x = -1, 7.$$

The  $y$ -intercept is  $y = 2(0 - 3)^2 - 32 = -14$ .