The Three Forms of a Quadratic Function (Parabola)

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General Form

$y = ax^2 + bx + c$

The *concavity* is determined by a.
If $a > 0$ the parabola is concave up.
If $a < 0$ the parabola is concave down.

The y-intercept is c.

The *axis of symmetry*, which is also the x-coordinate of the vertex, is $x = \frac{-b}{2a}$.

To find the x-intercepts, solve $0 = ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To get this form from the other forms:
Multiply it out & collect like terms.

Example
$y = 2x^2 - 12x - 14$
$a = 2$, $b = -12$, $c = -14$

The y-intercept is (0, -14).

The axis of symmetry is:

$$x = \frac{-(-12)}{(2)(2)} = 3. \text{ So } x = 3.$$

Factored form

$y = a\,(x - \alpha)\,(x - \beta)$

Gives the x-intercepts: $x = \alpha, \beta$.

To find the y-intercept set $x = 0$ and evaluate.

The *concavity* is determined by a.
If $a > 0$ the parabola is concave up.
If $a < 0$ the parabola is concave down.

The x-coordinate of the vertex and the equation of the axis of the symmetry is the <u>average</u> of the x-intercepts, $\frac{\alpha+\beta}{2}$.

To get this form from the other forms: Factor it.

Example
$y = 2(x + 1)\,(x - 7)$

The zeros are $x = -1, 7$

The x-intercepts are (-1, 0), (7, 0)

The axis of symmetry is:

$x = \frac{-1+7}{2}$, so $x = 3$.

The y-intercept is $y = 2(0+1)(0-7) = -14$

Vertex (or Standard) form

$y = a(x - h)^2 + k$

Gives the vertex: (h, k).
(Note the minus sign on h.)

The axis of symmetry is $x = h$.

The *concavity* is determined by a.
If $a > 0$ the parabola is concave up.
If $a < 0$ the parabola is concave down.

To get this form from the other forms: Complete the square.

To find the x-intercepts set $y = 0$ and solve for x.

To find the y-intercept set $x = 0$ and evaluate.

Example
$y = 2(x - 3)^2 - 32$

The vertex is (3, -32)

The axis of symmetry is: $x = 3$

To find the x-intercept $0=2(x-3)^2-32$
So $32 = 2(x - 3)^2$, so $16 = (x - 3)^2$,
so $\pm 4 = x - 3$, so $x = -1, 7$.

The y-intercept is $y=2(0-3)^2-32=-14$

The Four "Quadratic Equations"

There are four ***<u>different</u>*** mathematical objects that are sometimes called by students the "Quadratic Equation". Try to keep them straight.

Quadratic Expression	$ax^2 + bx + c$	It can be factored, but it cannot be "solved", because it's not an equation and only equations can be solved.
Quadratic Equation	$0 = ax^2 + bx + c$	It can be solved, possibly by factoring it.
Quadratic Function	$y = ax^2 + bx + c$	It cannot be "solved." It can be graphed and the x-intercepts can be solved for, possibly by factoring it.
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	The solution of the quadratic equation.

Several statements about Quadratics

There are several ways of discussing the solutions of $0 = a\,(x - \alpha)\,(x - \beta)$. The following are all the same statement:

1. The **zeros, roots** or **solutions** of the quadratic equation are α & β.
2. The **factors** of the corresponding quadratic expression are $(x - \alpha)\,(x - \beta)$.
3. The **zeros** of the corresponding quadratic function are α & β. I.e. these are the values of x that make $y = 0$.
4. The **x-intercepts** of the corresponding quadratic function are $(\alpha, 0)$, $(\beta, 0)$.

Example: Given that the x-intercepts of $f(x)$ are 2 and -3, solve $f(x) = 0$. The answer is simply $x = 2$ and -3.