Polynomials

Dr. William J. Larson - http://MathsTutorGeneva.ch/

- Examples of algebraic expressions that are polynomials
  $3x^2 - 1$, $9x^3 - 7x^5 - 3x^2 + 6.181x - 9$, 5, $0.5x$, x^2
  $-\sqrt{3}\,x + 4$, $-x^3 + \pi x - 3$, $5x^3 + 37x^{25} - \frac{3}{7}x^2 + 3x - 7$
- A **term** is a number multiplied by a variable (or variables) or just a number or just a variable.
- Polynomials are an **algebraic sum** of one or more terms.
- Polynomials are algebraic **expressions**.
  f(x) = polynomial is an **equation**, specifically a **function**.
- The number in front of the variable is the **coefficient**.
- The **degree** is the power of the highest power term. It is usually called **n.** For an expression to be a polynomial n must be a positive integer or zero.
- The **leading coefficient** is the coefficient of the highest power term.
- Polynomials with one, two and three terms are given special names: **monomials, binomials** and **trinomials** respectively.
- Polynomials are usually written in **standard form**, that is, with the terms arranged with descending powers:
  $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_3x^3 + a_2x^2 + a_1x^1 + a_0$.
  $a_n \neq 0$. All other coefficients could be 0.

Note that the subscript of the coefficient just keeps track of the power of its variable.

The number of terms, the degree & the leading coefficients of polynomials

	Number of terms	Degree = n	Leading Coefficient = a_n
$3x^2 - 1$	2	2	3
$9x^3 - \sqrt{2}\,x^5 - 3x^2 + 6.18x - 9$	5	5	$-\sqrt{2}$
$5 = 5x^0$	1	0	5
$0.5\,x = 0.5\,x^1$	1	1	½
$x^2 - 4x + 4$	3	2	1
$-x^3 + \pi x - 3$	3	3	-1
$5x^3 + 37x^{25} - 63x^5 - 96x^2 + 3x - 7$	6	25	37

Examples of algebraic expressions that are NOT polynomials:

$\frac{1}{x}$

$\frac{1}{(x-4)^2}$, x^{-2}

$x^{½} = \sqrt{x}$, $x^{⅔}$, $\sqrt[3]{x}$

$\sqrt{3x-7}$, $\frac{1}{x^2 - 3x + 3}$

$\sin x$, 2^x, e^{x-3}, $|x|$

$\log(x-3)$, $\ln(x^2 - 5x + 2)$

Features of All Polynomials:

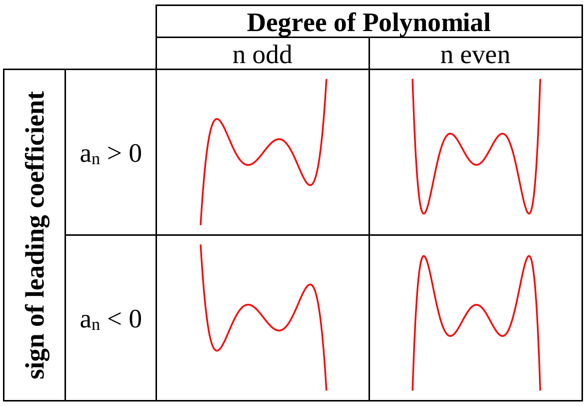
1. They are **continuous**. That is their graphs have no breaks. That is you can draw their graphs without picking up your pen.
2. Their graphs are **smooth**, that is, their graphs have **no sharp bends.**
3. The number of "**bumps**" is always less than n and greater than or equal to zero.

Calculus is needed to determine the actual number.
If n is odd, there are an even number of bumps.
If n is even, there are an odd number of bumps.
For example $f(x) = x^3$ has no bumps; $f(x) = x^3 - x$ has 2 bumps. ("Bumps" are also called **turning points** or **turns** or **local maxima & minima**.)

The Leading Coefficient Test (LHB & RHB)

The left and the right hand behavior (LHB & RHB) of a polynomial function means what happens to y = f(x) as x becomes very large - either positive (RHB) or negative – (LHB). The LHB & RHB of f(x) is determined by the leading term, that is by the $a_n x^n$ term. As x becomes very large, the polynomial always goes to $y = +\infty$ or to $y = -\infty$. (except for a zero degree polynomial, e.g. y = 4, which is simply a horizontal line)

1. If n is **even,** the LHB & RHB is the same.
   I.e. both are up or both are down.
2. If n is **odd,** LHB & RHB is opposite.
   I.e. one is up and the other is down.
3. If $a_n > 0$ as $x \to +\infty$, $f(x) \to +\infty$. That is it goes up.
   If $a_n < 0$ as $x \to +\infty$, $f(x) \to -\infty$. That is it goes down.



Four equivalent statements relating, f(x), 0 = f(x), y = f(x) & its graph

1. x = a is a **solution** or **root** of the *polynomial equation* f(x) = 0.
2. x = a is a **zero** of the *polynomial expression*, f(x).
3. (x – a) is a **factor** of the *polynomial expression* f(x). Note the minus sign.
4. (a, 0) is an **x-intercept** of the graph of the *polynomial function* y = f(x).