The Properties of Logarithms

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The definition of logarithms: $y = \log_a x \Leftrightarrow a^y = x \quad (a > 0, a \neq 1)$

This means that $y = \log_a x$ and $y = a^x$ are inverses.

$\log_b x$ is **pronounced "log base b of x"**

The Natural Logarithm = $\ln x \equiv \log_e x$

ln x is pronounced "el en x". ln stands for **n**atural **l**ogarithm. $e \cong 2.718281828459\ldots$

The Common Logarithm: $\log x \equiv \log_{10} x$

Elementary Properties of Logarithms

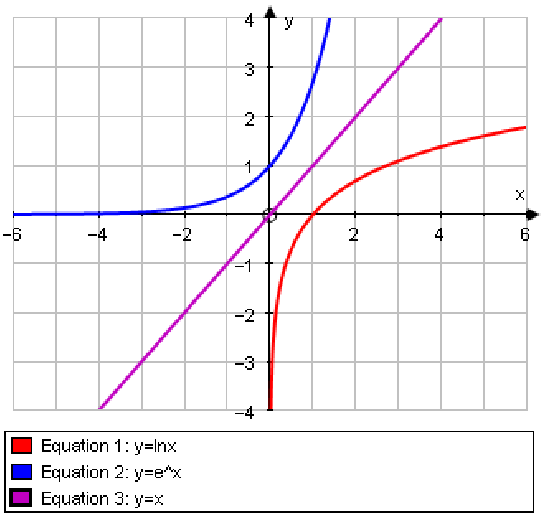
$\log_a x$ laws	log x laws	ln x laws
$\log_a 1 = 0$	$\log 1 = 0$	$\ln 1 = 0$
$\log_a a = 1$	$\log 10 = 1$	$\ln e = 1$
$\log_a a^x = x$	$\log 10^x = x$	$\ln e^x = x$
$a^{\log_a x} = x$	$10^{\log x} = x$	$e^{\ln x} = x$

The Laws of Logarithms

Law	because
1. $\log_a(u \times v) = \log_a u + \log_a v$	$a^u \times a^v = a^{u+v}$
2. $\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$	$\frac{a^u}{a^v} = a^{u-v}$
3. $\log_a u^x = x \log_a u$	$(a^x)^n = a^{xn}$
4. $\log_a c = \frac{\log_b c}{\log_b a}$	

Solving Equations with Logarithms & Exponents

1. $\log_a f(x) = \log_a g(x) \Leftrightarrow f(x) = g(x)$, $a > 0, a \neq 1$
2. $a^{f(x)} = a^{g(x)} \Leftrightarrow f(x) = g(x), a > 0, a \neq 1$
3. $f(x)^a = g(x)^a \Leftrightarrow f(x) = g(x)$



Log functions and exponential functions are inverses.
So they are reflections of each other across the line $y = x$.

	$y = \log_b x$	$y = b^x$
asymptote	vertical: $x = 0$	horizontal: $y = 0$
axis intercept	$x = 1$	$y = 1$