How to Draw Reciprocal Functions

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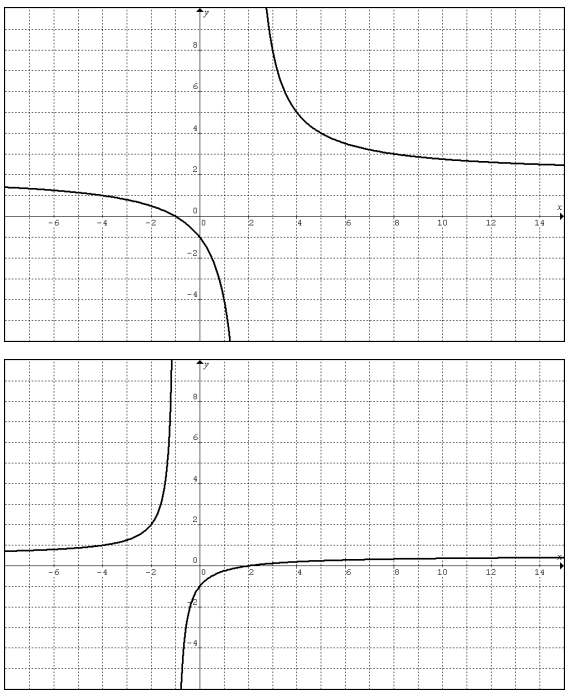
If $f(x) = \frac{1}{g(x)} = (g(x))^{-1}$, then f(x) and g(x) are reciprocal functions. Note that $g(x) = \frac{1}{f(x)}$.

Reciprocals are **not** inverses, i.e. $(f(x))^{-1} \neq f^{-1}(x)$.

Example If $f(x) = x^2$ and $g(x) = \frac{1}{x^2}$, f(x) & g(x) are reciprocal functions.

1. Where g(x) has a **zero**, f(x) has a **vertical asymptote**,
   because if g(x) approaches 0, f(x) approaches "1/0" i.e. ∞.
   And vice versa, i.e., where g(x) has a **vertical asymptote**, f(x) has a **zero**,
   because if g(x) approaches ∞, f(x) approaches "1/∞" i.e. 0.
2. Calculate and **plot any minima or maxima**.
   Example: If g(x) has a maximum at (3, 2), then f(x) will have a minimum at (3, 0.5).
3. If g(x) goes through 1, f(x) also **goes through 1**. Plot the point.
   Example: If g(x) goes through the point (4, 1), then f(x) will also go through the point (4, 1).
4. **Horizontal Asymptotes**
   a) If g(x) has a **horizontal asymptote** at y = a, then f(x) will have a horizontal asymptote at y = 1/a.
   b) If g(x) has a **horizontal asymptote at y = 0** (the x-axis), f(x) will not have a horizontal asymptote.
   c) If g(x) **does not have a horizontal asymptote**, f(x) will have a horizontal asymptote at y = 0 (the x-axis).
5. If g(x) > 0, then f(x) > 0, because 1 divided by a **positive** number is a positive number.
   If g(x) < 0, then f(x) < 0, because 1 divided by a **negative** number is a negative number.
6. Draw a curve through the points plotted, respecting the above rules.

A function and its reciprocal



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