Finding Local Maxima, Minima and Inflexion Points

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Local Maxima & Minima

Local maxima & minima (turning points) can occur (but might not occur) where:

1. $y' = 0$, which is called a **stationary point**.
2. y' is undefined
   e.g. at $x = 0$ for $y = x^{2/3}$ (a **cusp**)
   The IB exam ignores this case.
3. An endpoint of the domain.
   e.g. at $x = 0$ & 2 for $y = 3x + 2$, $0 \le x \le 2$.

$y' = 0$ locates **local** maxima & minima as opposed to **global** maxima & minima. There can be several local maxima or minima, but there can only be one global maxima or minima. Global max/min are often at $\pm\infty$, which $y' = 0$ will not find. (Technically since the graph never reaches $\pm\infty$ it's **not** a global max/min, but the IB does not test this.)

To find the local maxima & minima use any of:

Analytically (To "justify")

1st Derivative Test

Construct a sign table for y'. On each side of points where $y' = 0$ (or y is undefined) check whether y' is positive or negative.

To visualize this draw this as an upward or downward sloping line on the sign table.

1. If y' changes from + to – at a point, there is a local maximum there.
2. If y' changes from – to + at a point, there is a local minimum there.
3. If y' does not change sign, there is a horizontal inflexion point there.

2nd Derivative Test

At the x values where $y' = 0$, check whether y'' is positive or negative.

1. If $y'' < 0$ at such a point, there is a local maximum there.
2. If $y'' > 0$ at such a point, there is a local minimum there.

Graphically

Sketch the graph of y. Locate min/max visually. Use CALC minimum or maximum to find them.

Be aware that this may not give an exact answer, if one is required.

"My calculator said so", does not get the "justify" point.

Inflexion Points

Inflexion points are points on the curve where the concavity changes. The concavity can change (but might not change) where:

1. $y'' = 0$.
2. y'' is undefined.
   For example at $x = 1$ for $y = x^{5/3} + 5x^{2/3}$. $x = 1$ is an inflexion point.
   For $f(x) = \frac{1}{(x-1)^2}$, The concavity changes at $x = 1$, but $x = 1$ is **not** an inflexion point, because f(x) is not defined there. The IB exam ignores this case.

To find the inflexion points use any of:

Analytically (To "justify")

2nd Derivative Test

Check y'' on each side of the x values where $y'' = 0$.
Construct a sign table for y'' or test convenient values of x.
If y'' changes sign on opposite sides of the x value where $y'' = 0$:
If $f'(x) = 0$ there, it is a horizontal inflexion point.
If $f'(x) \neq 0$ there, it is a non-horizontal inflexion point.
To visualize this if $y'' > 0$ draw a concave up curve and if $y'' < 0$ draw a concave down curve.

Graphically

Graph y in Y1 in Y=. Turn it off.
Graph y' (**not** y) using ALPHA F2 3: nDeriv(and ALPHA F4
Set xRes = 3 (in WINDOW) to speed it up.

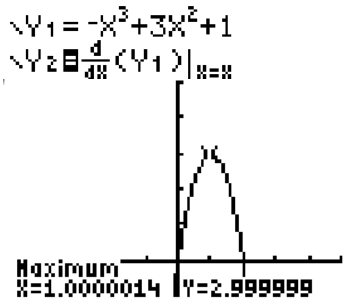
Use CALC minimum or maximum to find the local maxima & minima of y'.

Either a local maxima or minima of y' indicates an inflexion point.

Be aware that this will not give an exact answer, if one is required.

Example

Find the inflexion point of $y = -x^3 + 3x + 1$



Since y' has a maximum at $x = 1$, y has an inflexion point at x = 1

"My calculator said so", does not get the "justify" point.