Finding Local Maxima, Minima and Inflexion Points

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Local Maxima and Minima

Local maxima & minima (turning points) can occur (but might not occur) :

1. Where $y' = 0$, which is called a **stationary point**.

2. At an endpoint of the domain. e.g. at $x = 0$ & 2 for $y = 3x^2 + 2$, $0 \le x \le 2$.

Solving for the x values where $y' = 0$ locates **local** maxima & minima as opposed to **global** maxima & minima.
There can be several local maxima or minima, but there can only be one global maxima or minima.
Global maxima and minima are often at $\pm\infty$, which setting $y' = 0$ will not find, but the IB does not ask for either.

A point where $y' = 0$, could be a maximum, a minimum or a horizontal inflection point.
Sometimes you are required to "justify" which of these it is.

To justify the local maxima & minima you can do either of the:

1st Derivative Test

Construct a sign table for y'. I.e. on each side of the x values where $y' = 0$, check whether y' is positive or negative.

To visualize this draw an upward ($y' > 0$) or downward ($y' < 0$) sloping line on the sign table.

1. If y' changes from + to – at such a point, there is a local maximum there.
2. If y' changes from – to + at such a point, there is a local minimum there.
3. If y' does not change sign at such a point, there is a horizontal inflexion point there.

2nd Derivative Test

At the x values where $y' = 0$, check whether y'' is positive or negative.

1. If $y'' < 0$ at such a point, there is a local maximum there.
2. If $y'' > 0$ at such a point, there is a local minimum there.

Inflexion Points

Inflexion points are points on the curve where the concavity changes.
The concavity usually changes (but might not change) where: $y'' = 0$.

To find the inflexion points:

Check y'' on each side of the x values where $y'' = 0$.

Construct a sign table for y'' by testing convenient values of x.

If y'' changes sign on opposite sides of the x value where $y'' = 0$ and:

If $f'(x) = 0$ there, it is a horizontal inflexion point.

If $f'(x) \neq 0$ there, it is a normal inflexion point.

To visualize this if $y'' > 0$, draw a concave up curve and if $y'' < 0$, draw a concave down curve.