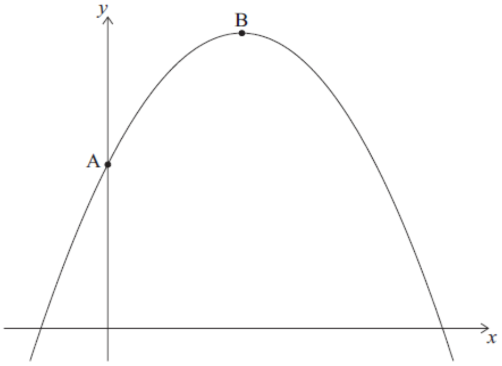
Studies Functions 2008-14 with MS

1a. *[1 mark]*

The graph of the quadratic function $f(x) = c + bx - x^2$ intersects the *y*-axis at point A(0, 5) and has its vertex at point B(2, 9).



Write down the value of *c*.

1b. *[2 marks]*

Find the value of *b*.

1c. *[2 marks]*

Find the *x*-intercepts of the graph of *f* .

1d. *[1 mark]*

Write down $f(x)$ in the form $f(x) = -(x - p)(x + q)$.

2a. *[1 mark]*

The number of bacteria in a colony is modelled by the function

$N(t) = 800 \times 3^{0.5t},\ t \geqslant 0$,

where N is the number of bacteria and t is the time in hours.

Write down the number of bacteria in the colony at time $t = 0$.

2b. *[3 marks]*

Calculate the number of bacteria present at 2 hours and 30 minutes. Give your answer correct to the nearest hundred bacteria.

2c. *[2 marks]*

Calculate the time, in hours, for the number of bacteria to reach 5500.

3a. *[1 mark]*

In a trial for a new drug, scientists found that the amount of the drug in the bloodstream decreased over time, according to the model

$D(t) = 1.2 \times (0.87)^t,\ t \geqslant 0$

where D is the amount of the drug in the bloodstream in mg per litre $(\text{mg}\,\text{l}^{-1})$ and t is the time in hours.

Write down the amount of the drug in the bloodstream at $t = 0$.

3b. *[2 marks]*

Calculate the amount of the drug in the bloodstream after 3 hours.

3c. *[3 marks]*

Use your graphic display calculator to determine the time it takes for the amount of the drug in the bloodstream to decrease to $0.333\ \text{mg1}^{-1}$.

4a. *[1 mark]*

A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm.

The total volume of the parcel is 3000c m^3.

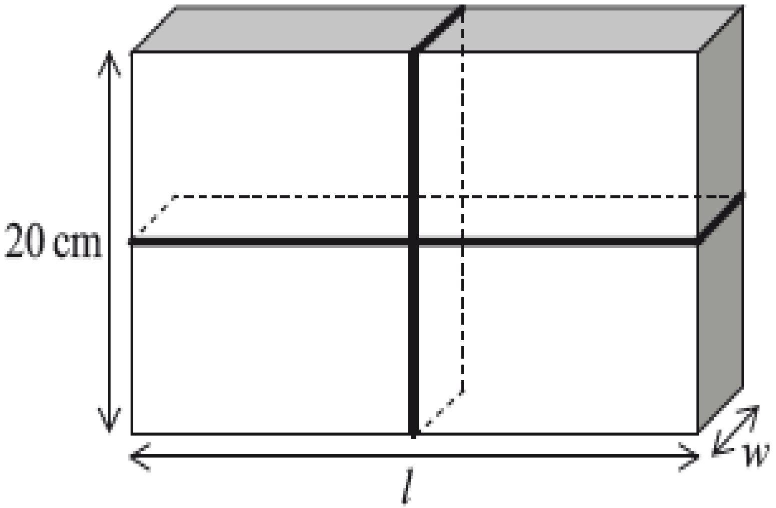
Express the volume of the parcel in terms of l and w.

4b. *[2 marks]*

Show that $l = \frac{150}{w}$.

4c. *[2 marks]*

The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram.



Show that the length of string, S cm, required to tie up the parcel can be written as

$$S = 40 + 4w + \frac{300}{w}, \ 0 < w \leqslant 20.$$

4d. *[2 marks]*

The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram.

Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

4e. *[3 marks]*

The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram.

Find $\frac{\mathrm{d}S}{\mathrm{d}w}$.

4f. *[2 marks]*

The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram.

Find the value of w for which S is a minimum.

4g. *[1 mark]*

The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram.

Write down the value, l, of the parcel for which the length of string is a minimum.

4h. *[2 marks]*

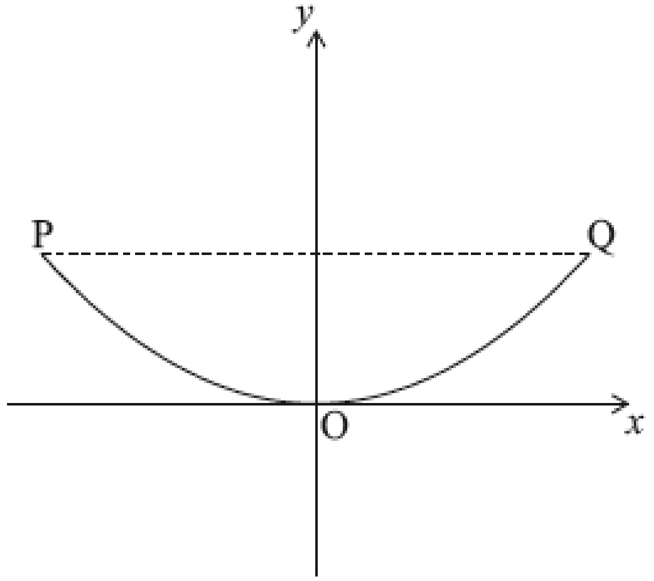
The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram.

Find the minimum length of string required to tie up the parcel.

5a. *[1 mark]*

The front view of the edge of a water tank is drawn on a set of axes shown below.

The edge is modelled by $y = ax^2 + c$.



Point P has coordinates $(-3, 1.8)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(3, 1.8)$.

Write down the value of c.

5b. *[2 marks]*

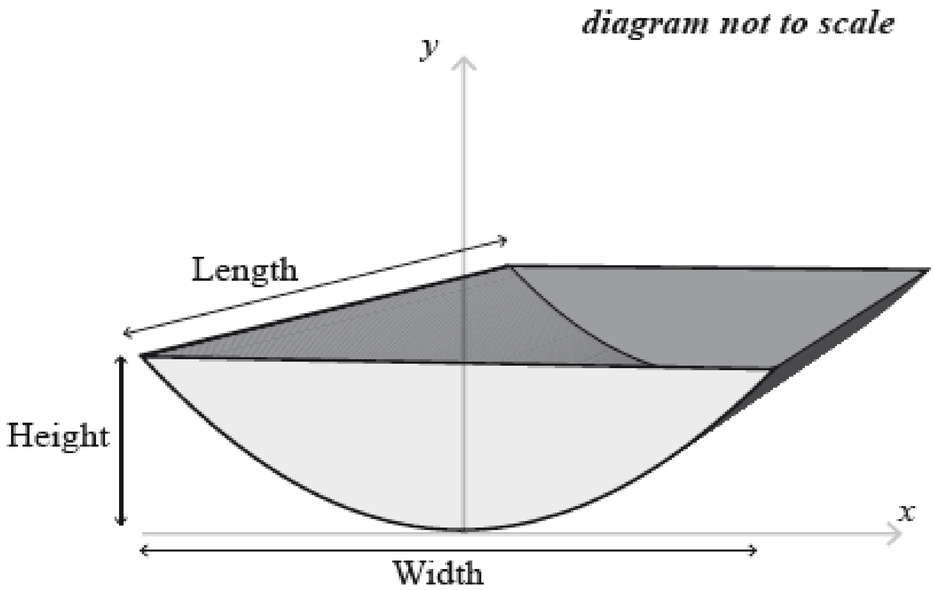
Find the value of a.

5c. *[1 mark]*

Hence write down the equation of the quadratic function which models the edge of the water tank.

5d. *[2 marks]*

The water tank is shown below. It is partially filled with water.



Calculate the value of *y* when $x = 2.4\mathrm{m}$.

5e. *[2 marks]*

The water tank is shown below. It is partially filled with water.

State what the value of x and the value of y represent for this water tank.

5f. *[2 marks]*

The water tank is shown below. It is partially filled with water.

Find the value of x when the height of water in the tank is 0.9 m.

5g. *[2 marks]*

The water tank is shown below. It is partially filled with water.

When the water tank is filled to a height of 0.9 m, the front cross-sectional area of the water is $2.55\ \mathrm{m}^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is $36\ \mathrm{m}^3$.

(ii) Calculate the percentage of water in the tank.

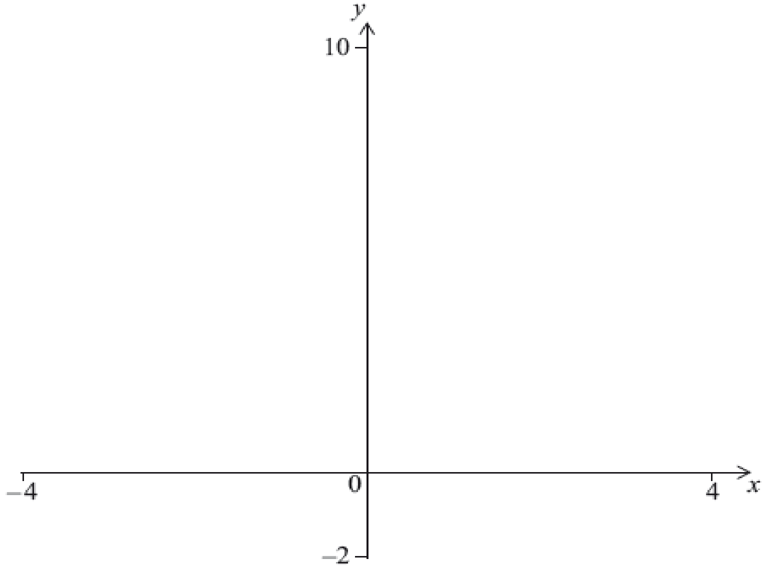
6a. *[4 marks]*

Consider the two functions, f and g, where

$f(x) = \frac{5}{x^2+1}$

$g(x) = (x-2)^2$

Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the axes below. Indicate clearly the points where each graph intersects the y-axis.

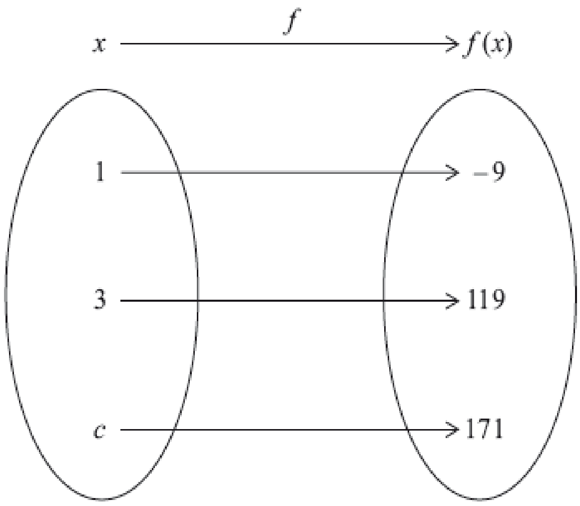


6b. *[2 marks]*

Use your graphic display calculator to solve $f(x) = g(x)$.

7a. *[2 marks]*

A quadratic function $f : x \mapsto ax^2 + b$, where a and $b \in \mathbb{R}$ and $x \geqslant 0$, is represented by the mapping diagram.



Using the mapping diagram, write down two equations in terms of a and b.

7b. *[2 marks]*

Solve the equations to find the value of

(i) a;

(ii) b.

7c. *[2 marks]*

Find the value of c.

8a. *[2 marks]*

A computer virus spreads according to the exponential model

$N = 200 \times (1.9)^{0.85t},\ t \geqslant 0$

where N is the number of computers infected, and t is the time, in hours, after the initial infection. Calculate the number of computers infected after 6 hours.

8b. *[4 marks]*

Calculate the time for the number of infected computers to be greater than $1\,000\,000$.

Give your answer correct to the nearest hour.

9a. *[2 marks]*

Consider the function $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 20$.

Find $f(-2)$.

9b. *[3 marks]*

Find $f'(x)$.

9c. *[5 marks]*

The graph of the function $f(x)$ has a local minimum at the point where $x = -2$.

Using your answer to part (b), show that there is a second local minimum at $x = 3$.

9d. *[4 marks]*

The graph of the function $f(x)$ has a local minimum at the point where $x = -2$.

Sketch the graph of the function $f(x)$ for $-5 \leqslant x \leqslant 5$ and $-40 \leqslant y \leqslant 50$. Indicate on your sketch the coordinates of the y-intercept.

9e. *[2 marks]*

The graph of the function $f(x)$ has a local minimum at the point where $x = -2$.

Write down the coordinates of the local maximum.

9f. *[2 marks]*

Let T be the tangent to the graph of the function $f(x)$ at the point $(2, -12)$.

Find the gradient of T.

9g. *[5 marks]*

The line L passes through the point $(2, -12)$ and is perpendicular to T.

L has equation $x + by + c = 0$, where b and $c \in \mathbb{Z}$.

Find

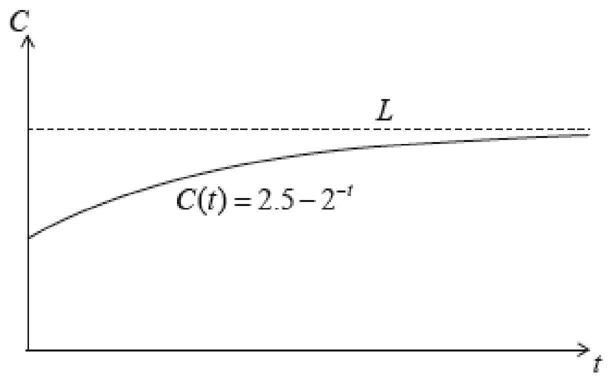
(i) the gradient of L;

(ii) the value of b and the value of c.

10a. *[1 mark]*

The amount of electrical charge, C, stored in a mobile phone battery is modelled by $C(t) = 2.5 - 2^{-t}$, where t, in hours, is the time for which the battery is being charged.

diagram not to scale



Write down the amount of electrical charge in the battery at $t = 0$.

10b. *[2 marks]*

The line L is the horizontal asymptote to the graph.

Write down the equation of L.

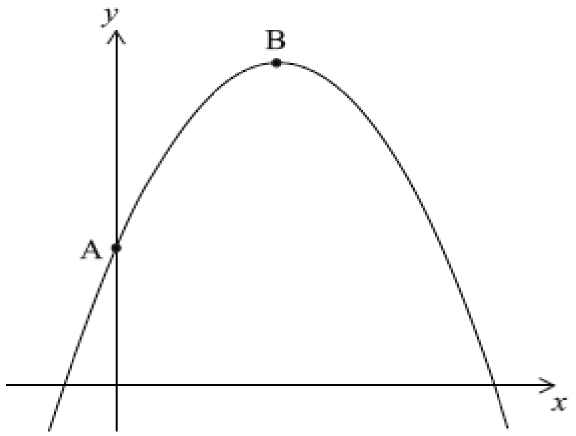
10c. *[3 marks]*

To download a game to the mobile phone, an electrical charge of 2.4 units is needed.

Find the time taken to reach this charge. Give your answer correct to the nearest minute.

11a. *[1 mark]*

The graph of the quadratic function $f(x) = ax^2 + bx + c$ intersects the y-axis at point A (0, 5) and has its vertex at point B (4, 13).



Write down the value of c.

11b. *[3 marks]*

By using the coordinates of the vertex, B, or otherwise, write down **two** equations in a and b.

11c. *[2 marks]*

Find the value of a and of b.

12a. *[3 marks]*

Consider the sequence $u_1,\ u_2,\ u_3,\ \ldots,\ u_n,\ \cdots$ where

$u_1 = 600,\ u_2 = 617,\ u_3 = 634,\ u_4 = 651.$

The sequence continues in the same manner.

Find the value of u_{20}.

12b. *[3 marks]*

Find the sum of the first 10 terms of the sequence.

12c. *[3 marks]*

Now consider the sequence $v_1,\ v_2,\ v_3,\ \ldots,\ v_n,\ \cdots$ where

$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$

This sequence continues in the same manner.

Find the exact value of v_{10}.

12d. *[3 marks]*

Now consider the sequence $v_1,\ v_2,\ v_3,\ \ldots,\ v_n,\ \cdots$ where

$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$

This sequence continues in the same manner.

Find the sum of the first 8 terms of this sequence.

12e. *[3 marks]*

k is the smallest value of n for which v_n is greater than u_n.

Calculate the value of k.

Studies Functions 2008-14 MS

1a. *[1 mark]*

Markscheme

5 ***(A1) (C1)***

1b. *[2 marks]*

Markscheme

$\frac{-b}{2(-1)} = 2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in axis of symmetry formula.

OR

$y = 5 + bx - x^2$

$9 = 5 + b(2) - (2)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 9 and 2 into their quadratic equation.

$(b =)4$ ***(A1)*(ft) *(C2)***

Note: Follow through from part (a).

1c. *[2 marks]*

Markscheme

5, –1 ***(A1)*(ft)*(A1)*(ft) *(C2)***

Notes: Follow through from parts (a) and (b), irrespective of working shown.

1d. *[1 mark]*

Markscheme

$f(x) = -(x - 5)(x + 1)$ ***(A1)*(ft) *(C1)***

Notes: Follow through from part (c).

Examiners report

Many candidates did not see the connection between the *x*-intercepts and the factored form of a quadratic function. The syllabus explicitly sates that the graphs of quadratics should be linked to solutions of quadratic equations by factorizing and vice versa. This was one of the most challenging questions for candidates.

2a. *[1 mark]*

Markscheme

800 ***(A1) (C1)***

Examiners report

[N/A]

2b. *[3 marks]*

Markscheme

$800 \times 3^{(0.5 \times 2.5)}$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted formula.

$= 3158.57$... ***(A1)***

$= 3200$ ***(A1) (C3)***

Notes: Final ***(A1)*** is given for correctly rounding ***their*** answer. This may be awarded regardless of a preceding ***(A0)***.

Examiners report

[N/A]

2c. *[2 marks]*

Markscheme

$5500 = 800 \times 3^{(0.5 \times t)}$ ***(M1)***

Notes: Award ***(M1)*** for equating function to 5500. Accept correct alternative methods.

$= 3.51 \text{ hours}$ (3.50968...) ***(A1) (C2)***

Examiners report

[N/A]

3a. *[1 mark]*

Markscheme

$1.2\ (\text{mg l}^{-1})$ ***(A1) (C1)***

[1 mark]

Examiners report

[N/A]

3b. *[2 marks]*

Markscheme

$1.2 \times (0.87)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into given formula.

$= 0.790\ (\text{mg l}^{-1})\ (0.790203\ldots)$ ***(A1) (C2)***

[2 marks]

Examiners report

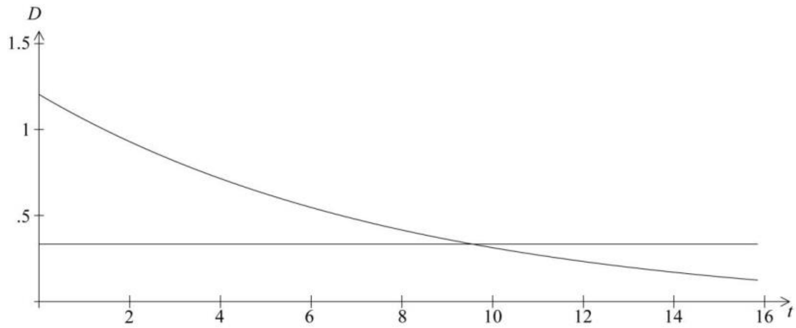
[N/A]

3c. *[3 marks]*

Markscheme

$1.2 \times 0.87^t = 0.333$ ***(M1)***

Note: Award ***(M1)*** for setting up the equation.



(M1)

Notes: Some indication of scale is to be shown, for example the window used on the calculator.
Accept alternative methods.

9.21 (hours) ($9.20519\ldots$, 9 hours 12 minutes, 9:12) ***(A1) (C3)***

[3 marks]

Examiners report

[N/A]

4a. *[1 mark]*

Markscheme

$20lw$ **OR** $V = 20lw$ ***(A1)***

[1 mark]

Examiners report

[N/A]

4b. *[2 marks]*

Markscheme

$3000 = 20lw$ ***(M1)***

Note: Award ***(M1)*** for equating their answer to part (a) to 3000.

$l = \frac{3000}{20w}$ ***(M1)***

Note: Award ***(M1)*** for rearranging equation to make l subject of the formula. The above equation must be seen to award ***(M1)***.

OR

$150 = lw$ ***(M1)***

Note: Award ***(M1)*** for division by 20 on both sides. The above equation must be seen to award ***(M1)***.

$l = \frac{150}{w}$ ***(AG)***

[2 marks]

Examiners report

[N/A]

4c. *[2 marks]*

Markscheme

$S = 2l + 4w + 2(20)$ ***(M1)***

Note: Award ***(M1)*** for setting up a correct expression for S.

$2\left(\frac{150}{w}\right) + 4w + 2(20)$ ***(M1)***

Notes: Award ***(M1)*** for correct substitution into the expression for S. The above expression must be seen to award ***(M1)***.

$= 40 + 4w + \frac{300}{w}$ ***(AG)***

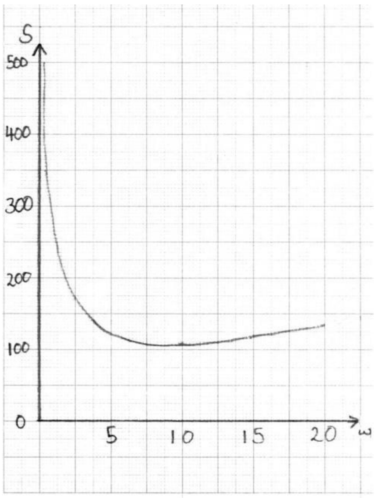
[2 marks]

Examiners report

[N/A]

4d. *[2 marks]*

Markscheme



(A1)(A1)(A1)(A1)

Note: Award ***(A1)*** for correct scales, window and labels on axes, ***(A1)*** for approximately correct shape, ***(A1)*** for minimum point in approximately correct position, ***(A1)*** for asymptotic behaviour at $w = 0$.
Axes must be drawn with a ruler and labeled w and S.
For a smooth curve (with approximately correct shape) there should be **one** continuous thin line, no part of which is straight and no (one-to-many) mappings of w.
The S-axis must be an asymptote. The curve must not touch the S-axis nor must the curve approach the asymptote then deviate away later.
[4 marks]
Examiners report
[N/A]

4e. *[3 marks]*
Markscheme
$4 - \frac{300}{w^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for 4, ***(A1)*** for -300, ***(A1)*** for $\frac{1}{w^2}$ or w^{-2}. If extra terms present, award at most ***(A1)(A1)(A0)***.
[3 marks]
Examiners report
[N/A]

4f. *[2 marks]*
Markscheme
$4 - \frac{300}{w^2} = 0$ **OR** $\frac{300}{w^2} = 4$ **OR** $\frac{\mathrm{d}S}{\mathrm{d}w} = 0$ ***(M1)***
Note: Award ***(M1)*** for equating their derivative to zero.
$w = 8.66 \ \left(\sqrt{75},\ 8.66025\ldots\right)$ ***(A1)*(ft)*(G2)***
Note: Follow through from their answer to part (e).
[2 marks]
Examiners report
[N/A]

4g. *[1 mark]*
Markscheme
$17.3\left(\frac{150}{\sqrt{75}},\ 17.3205\ldots\right)$ ***(A1)*(ft)**
Note: Follow through from their answer to part (f).
[1 mark]
Examiners report
[N/A]

4h. *[2 marks]*
Markscheme
$40 + 4\sqrt{75} + \frac{300}{\sqrt{75}}$ ***(M1)***
Note: Award ***(M1)*** for substitution of their answer to part (f) into the expression for S.
$= 110 \text{ (cm)} \ \left(40 + 40\sqrt{3},\ 109.282\ldots\right)$ ***(A1)*(ft)*(G2)***
Note: Do not accept 109.
Follow through from their answers to parts (f) and (g).
[2 marks]
Examiners report
[N/A]

5a. *[1 mark]*
Markscheme
0 ***(A1)(G1)***
[1 mark]
Examiners report
[N/A]

5b. *[2 marks]*
Markscheme
$1.8 = a(3)^2 + 0$ ***(M1)***
OR
$1.8 = a(-3)^2 + 0$ ***(M1)***
Note: Award ***(M1)*** for substitution of $y = 1.8$ or $x = 3$ and their value of c into equation. 0 may be implied.
$a = 0.2\left(\frac{1}{5}\right)$ ***(A1)*(ft)*(G1)***
Note: Follow through from their answer to part (a).
Award ***(G1)*** for a correct answer only.
[2 marks]
Examiners report
[N/A]

5c. *[1 mark]*

Markscheme

$y = 0.2x^2$ ***(A1)*(ft)**

Note: Follow through from their answers to parts (a) and (b).
Answer must be an equation.

[1 mark]

Examiners report

[N/A]

5d. *[2 marks]*

Markscheme

$0.2 \times (2.4)^2$ ***(M1)***

$= 1.15\ (\mathrm{m})\ (1.152)$ ***(A1)*(ft)*(G1)***

Notes: Award ***(M1)*** for correctly substituted formula, ***(A1)*** for correct answer. Follow through from their answer to part (c).
Award ***(G1)*** for a correct answer only.

[2 marks]

Examiners report

[N/A]

5e. *[2 marks]*

Markscheme

y is the height ***(A1)***

positive value of x is half the width (*or equivalent*) ***(A1)***

[2 marks]

Examiners report

[N/A]

5f. *[2 marks]*

Markscheme

$0.9 = 0.2x^2$ ***(M1)***

Note: Award ***(M1)*** for setting their equation equal to 0.9.

$x = \pm 2.12\ (\mathrm{m})\ \left(\pm \frac{3}{2}\sqrt{2},\ \pm\sqrt{4.5},\ \pm 2.12132\ldots\right)$ ***(A1)*(ft)*(G1)***

Note: Accept 2.12. Award ***(G1)*** for a correct answer only.

[2 marks]

Examiners report

[N/A]

5g. *[2 marks]*

Markscheme

(i) 2.55×5 ***(M1)***

Note: Award ***(M1)*** for correct substitution in formula.

$= 12.8(\ \mathrm{m}^3)\ \left(12.75\ (\mathrm{m}^3)\right)$ ***(A1)(G2)***

[2 marks]

(ii) $\frac{12.75}{36} \times 100$ ***(M1)***

Note: Award ***(M1)*** for correct quotient multiplied by 100.

$= 35.4(\%)\ (35.4166\ldots)$ ***(A1)*(ft)*(G2)***

Note: Award ***(G2)*** for $35.6(\%)(35.5555\ldots(\%))$.
Follow through from their answer to part (g)(i).

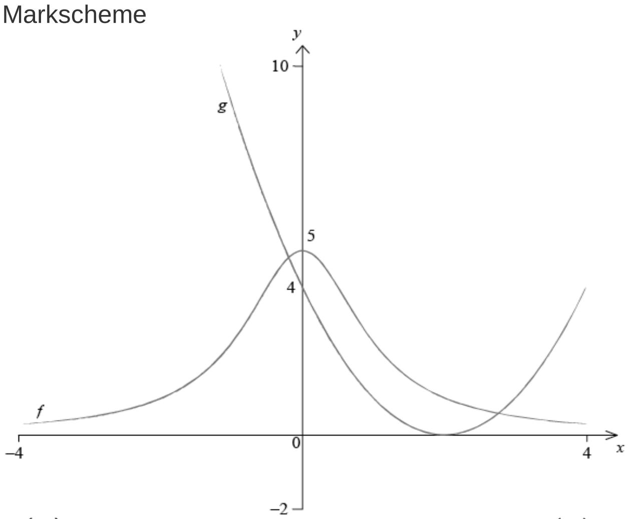
[2 marks]

Examiners report

[N/A]

6a. *[4 marks]*

Markscheme



$f(x)$: a smooth curve symmetrical about *y*-axis, $f(x) > 0$ ***(A1)***

Note: If the graph crosses the *x*-axis award ***(A0)***.
Intercept at their numbered $y = 5$ ***(A1)***
Note: Accept clear scale marks instead of a number.
$g(x)$: a smooth parabola with axis of symmetry at about $x = 2$ (the 2 does not need to be numbered) and $g(x) \geqslant 0$ ***(A1)***
Note: Right hand side must not be higher than the maximum of $f(x)$ at $x = 4$.
Accept the quadratic correctly drawn beyond $x = 4$.
Intercept at their numbered $y = 4$ ***(A1) (C4)***
Note: Accept clear scale marks instead of a number.
[4 marks]

6b. *[2 marks]*
Markscheme
$-0.195, 2.76$ $(-0.194808\ldots, 2.761377\ldots)$ ***(A1)*(ft)*(A1)*(ft) *(C2)***
Note: Award ***(A0)(A1)*(ft)** if both coordinates are given.
Follow through only if $f(x) = \frac{5}{x^2} + 1$ is sketched; the solutions are $-0.841, 3.22$ $(-0.840913\ldots, 3.217747...)$
[2 marks]
Examiners report
Many candidates attempted this question but relatively few were awarded the full six marks. Although they were asked to indicate clearly where the graph met the axes, many did not do this. Some entered the functions incorrectly into their calculator. A common error in part (b) was to give ordered pairs and therefore were not awarded the final mark.

7a. *[2 marks]*
Markscheme
$a(1)^2 + b = -9$ ***(A1)***
$a(3)^2 + b = 119$ ***(A1) (C2)***
Note: Accept equivalent forms of the equations.
[2 marks]
Examiners report

7b. *[2 marks]*
Markscheme
(i) $a = 16$ ***(A1)*(ft)**
(ii) $b = -25$ ***(A1)*(ft) *(C2)***
Note: Follow through from part (a) irrespective of whether working is seen.
If working is seen follow through from part (i) to part (ii).
[2 marks]

7c. *[2 marks]*
Markscheme
$16c^2 - 25 = 171$ ***(M1)***
Note: Award ***(M1)*** for correct quadratic with their a and b substituted.
$c = 3.5$ ***(A1)*(ft) *(C2)***
Note: Accept x instead of c.
Follow through from part (b).
Award ***(A1)*** only, for an answer of ± 3.5 with or without working.
[2 marks]
Examiners report
This question was answered reasonably well with many candidates able to write down the two equations and solve them for a and b. Errors such as mistaking the equation given for $3a^2 + b = 119$ meant that marks were lost even though the candidates appeared to know what they needed to do. Most candidates who were able to set up the equation in part (c) solved it correctly. Follow through marks were awarded to many candidates for correct working with their substituted values from part (b).

8a. *[2 marks]*
Markscheme
$200 \times (1.9)^{0.85 \times 6}$ ***(M1)***
Note: Award ***(M1)*** for correct substitution into given formula.
$= 5280$ ***(A1) (C2)***
Note: Accept 5281 or 5300 but no other answer.
[2 marks]
Examiners report
This question was answered very well, although some candidates were not awarded the final mark because the answer was not an integer number of computers. In part (b), some candidates neglected to give their answer correct to the nearest hour and lost the final mark.

8b. *[4 marks]*
Markscheme
$1\,000\,000 < 200 \times (1.9)^{0.85t}$ ***(M1)(M1)***

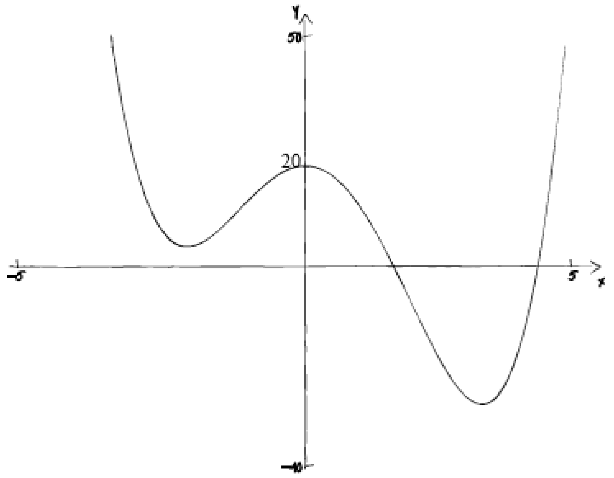
Note: Award ***(M1)*** for setting up the inequality (accept an equation), and ***(M1)*** for $1\,000\,000$ seen in the inequality or equation.
$t = 15.6\,(15.6113\ldots)$ ***(A1)***
16 hours ***(A1)*(ft)** ***(C4)***
Note: The final ***(A1)*(ft)** is for rounding **up** their answer to the nearest hour.
Award ***(C3)*** for an answer of 15.6 with no working.
Accept $1\,000\,001$ in an equation.
[4 marks]
Examiners report
This question was answered very well, although some candidates were not awarded the final mark because the answer was not an integer number of computers. In part (b), some candidates neglected to give their answer correct to the nearest hour and lost the final mark.

9a. *[2 marks]*
Markscheme
$\frac{3}{4}(-2)^4 - (-2)^3 - 9(-2)^2 + 20$ ***(M1)***
Note: Award ***(M1)*** for substituting $x = -2$ in the function.
$= 4$ ***(A1)(G2)***
Note: If the coordinates $(-2,\ 4)$ are given as the answer award, at most, ***(M1)(A0)***. If no working shown award ***(G1)***.
If $x = -2,\ y = 4$ seen then award full marks.
[2 marks]
Examiners report

9b. *[3 marks]*
Markscheme
$3x^3 - 3x^2 - 18x$ ***(A1)(A1)(A1)***
Note: Award ***(A1)*** for each correct term, award at most ***(A1)(A1)(A0)*** if extra terms seen.
[3 marks]
Examiners report

9c. *[5 marks]*
Markscheme
$f'(3) = 3 \times (3)^3 - 3 \times (3)^2 - 18 \times 3$ ***(M1)***
Note: Award ***(M1)*** for substitution in their $f'(x)$ of $x = 3$.
$= 0$ ***(A1)***
OR
$3x^3 - 3x^2 - 18x = 0$ ***(M1)***
Note: Award ***(M1)*** for equating their $f'(x)$ to zero.
$x = 3$ ***(A1)***
$f'(x_1) = 3 \times (x_1)^3 - 3 \times (x_1)^2 - 18 \times x_1 < 0$ where $0 < x_1 < 3$ ***(M1)***
Note: Award ***(M1)*** for substituting a value of x_1 in the range $0 < x_1 < 3$ into their f' and showing it is negative (decreasing).
$f'(x_2) = 3 \times (x_2)^3 - 3 \times (x_2)^2 - 18 \times x_2 > 0$ where $x_2 > 3$ ***(M1)***
Note: Award ***(M1)*** for substituting a value of x_2 in the range $x_2 > 3$ into their f' and showing it is positive (increasing).
OR
With or without a sketch:
Showing $f(x_1) > f(3)$ where $x_1 < 3$ and x_1 is close to 3. ***(M1)***
Showing $f(x_2) > f(3)$ where $x_2 > 3$ and x_2 is close to 3. ***(M1)***
Note: If a sketch of $f(x)$ is drawn **in this part of the question and** $x = 3$ is identified as a stationary point on the curve, then
(i) award, at most, ***(M1)(A1)(M1)(M0)*** if the stationary point has been found;
(ii) award, at most, ***(M0)(A0)(M1)(M0)*** if the stationary point has not been previously found.
Since the gradients go from negative (decreasing) through zero to positive (increasing) it is a local minimum ***(R1)(AG)***
Note: Only award ***(R1)*** if the first two marks have been awarded *ie* $f'(3)$ has been shown to be equal to 0.
[5 marks]

9d. *[4 marks]*
Markscheme



(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for labelled axes and indication of scale on both axes.
Award ***(A1)*** for smooth curve with correct shape.
Award ***(A1)*** for local minima in 2^{nd} and 4^{th} quadrants.
Award ***(A1)*** for *y* intercept $(0, 20)$ seen and labelled. Accept 20 on y-axis.
Do **not** award the third ***(A1)*** mark if there is a turning point on the x-axis.
If the derivative function is sketched then award, at most, ***(A1)(A0)(A0)(A0)***.
For a smooth curve (with correct shape) there should be **ONE** continuous thin line, no part of which is straight and no (one to many) mappings of x.
[4 marks]

9e. *[2 marks]*
Markscheme
$(0, 20)$ ***(G1)(G1)***
Note: If parentheses are omitted award ***(G0)(G1)***.
OR
$x = 0,\ y = 20$ ***(G1)(G1)***
Note: If the derivative function is sketched in part (d), award ***(G1)*****(ft)*****(G1)*****(ft)** for $(-1.12, 12.2)$.
[2 marks]

9f. *[2 marks]*
Markscheme
$f'(2) = 3(2)^3 - 3(2)^2 - 18(2)$ ***(M1)***
Notes: Award ***(M1)*** for substituting $x = 2$ into their $f'(x)$.
$= -24$ ***(A1)*****(ft)*****(G2)***
[2 marks]
Examiners report

9g. *[5 marks]*
Markscheme
(i) Gradient of perpendicular $= \frac{1}{24}\ (0.0417, 0.041666\ldots)$ ***(A1)*****(ft)*****(G1)***
Note: Follow through from part (f).
(ii) $y + 12 = \frac{1}{24}(x - 2)$ ***(M1)(M1)***
Note: Award ***(M1)*** for correct substitution of $(2, -12)$, ***(M1)*** for correct substitution of their perpendicular gradient into equation of line.
OR
$-12 = \frac{1}{24} \times 2 + d$ ***(M1)***
$d = -\frac{145}{12}$
$y = \frac{1}{24}x - \frac{145}{12}$ ***(M1)***
Note: Award ***(M1)*** for correct substitution of $(2, -12)$ and gradient into equation of a straight line, ***(M1)*** for correct substitution of the perpendicular gradient and correct substitution of $d$into equation of line.
$b = -24,\ c = -290$ ***(A1)*****(ft)*****(A1)*****(ft)*****(G3)***
Note: Follow through from parts (f) and g(i).
To award **(ft)** marks, b and c must be integers.
Where candidate has used 0.042 from g(i), award ***(A1)*****(ft)** for -288.
[5 marks]
Examiners report
Surprisingly, a correct method for substituting the value of –2 into the given function led many candidates to a variety of incorrect answers. This suggests a poor handling of negative signs and/or poor use of the graphic display calculator. Many correct answers were seen in part (b) as candidates seemed to be well-drilled in the process of differentiation. Part (c), however, proved to be quite a discriminator. There were 5 marks for this part of the question and simply showing that $x - 3$ is a turning point was not sufficient for all of these marks. Many simply

scored only two marks by substituting $x-3$ into their answer to part (b). Once they had shown that there was a turning point at $x-3$, candidates were not expected to use the second derivative but to show that the function decreases for $x<3$ and increases for $x>3$. Part (d) required a sketch which could have either been done on lined paper or on graph paper. The majority of candidates obtained at least two marks here with the most common errors seen being incomplete labelled axes and curves which were far from being smooth. In part (e), many candidates identified the correct coordinates for the two marks available. But for many candidates, this is where responses stopped as, in part (f), connecting the gradient function found in part (b) to the given coordinates proved problematic and only a significant minority of candidates were able to arrive at the required answer of –24. Indeed, there were many NR (no responses) to this part and the final part of the question. As many candidates found part (f) difficult, even more candidates found getting beyond the gradient of *L* very difficult indeed. A minority of candidates wrote down the gradient of their perpendicular but then did not seem to know where to proceed from there. Substituting their gradient for *b* and the coordinates (2, –12) into the equation $x+by+c=0$ was a popular, but erroneous, method. It was a rare event indeed to see a script with a correct answer for this part of the question.

10a. *[1 mark]*

Markscheme

1.5 ***(A1) (C1)***

[1 mark]

Examiners report

[N/A]

10b. *[2 marks]*

Markscheme

$C=2.5$ (accept $y=2.5$) ***(A1)(A1) (C2)***

Notes: Award ***(A1)*** for $C\ (\text{or } y)=$ a positive constant, ***(A1)*** for the constant $=2.5$.

Answer must be an equation.

[2 marks]

Examiners report

[N/A]

10c. *[3 marks]*

Markscheme

$2.4=2.5-2^{-t}$ ***(M1)***

Note: Award ***(M1)*** for setting the equation equal to 2.4 or for a horizontal line drawn at approximately $C=2.4$.

Allow x instead of t.

OR

$-t\ln(2)=\ln(0.1)$ ***(M1)***

$t=3.32192\dots$ ***(A1)***

$t=3\text{ hours and 19 minutes (199 minutes)}$ ***(A1)*(ft) *(C3)***

Note: Award the final ***(A1)*(ft)** for correct conversion of **their** time in hours to the nearest minute.

[3 marks]

Examiners report

[N/A]

11a. *[1 mark]*

Markscheme

5 ***(A1) (C1)***

[1 mark]

Examiners report

[N/A]

11b. *[3 marks]*

Markscheme

at least one of the following equations required

$a(4)^2+4b+5=13$

$4=-\frac{b}{2a}$

$a(8)^2+8b+5=5$ ***(A2)(A1) (C3)***

Note: Award ***(A2)(A0)*** for one correct equation, or its equivalent, and ***(C3)*** for any two correct equations.

Follow through from part (a).

The equation $a(0)^2+b(0)=5$ earns no marks.

[3 marks]

Examiners report

[N/A]

11c. *[2 marks]*

Markscheme

$a=-\frac{1}{2}$, $b=4$ ***(A1)*(ft)*(A1)*(ft) *(C2)***

Note: Follow through from their equations in part (b), but only if their equations lead to unique solutions for a and b.

[2 marks]

Examiners report

[N/A]

12a. *[3 marks]*

Markscheme

$600 + (20 - 1) \times 17$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic sequence formula, ***(A1)*** for correct substitutions. If a list is used, award ***(M1)*** for at least 6 correct terms seen, award ***(A1)*** for at least 20 correct terms seen.

$= 923$ ***(A1)(G3)***

[3 marks]

Examiners report

[N/A]

12b. *[3 marks]*

Markscheme

$\frac{10}{2}\left[2 \times 600 + (10 - 1) \times 17\right]$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic series formula, ***(A1)*** for their correct substitutions. Follow through from part (a). For consistent use of geometric series formula in part (b) with the geometric sequence formula in part (a) award a maximum of ***(M1)(A1)(A0)*** since their final answer cannot be an integer.

OR

$u_{10} = 600 + (10 - 1)17 = 753$ ***(M1)***

$S_{10} = \frac{10}{2}\left(600 + \text{their } u_{10}\right)$ ***(M1)***

Note: Award ***(M1)*** for their correctly substituted arithmetic sequence formula, ***(M1)*** for their correctly substituted arithmetic series formula. Follow through from part (a) and **within** part (b).

Note: If a list is used, award ***(M1)*** for at least 10 correct terms seen, award ***(A1)*** for these terms being added.

$= 6765$ (accept 6770) ***(A1)*****(ft)*****(G2)***

[3 marks]

Examiners report

[N/A]

12c. *[3 marks]*

Markscheme

3×2^9 ***(M1)(A1)***

Note: Award ***(M1)*** for substituted geometric sequence formula, ***(A1)*** for correct substitutions. If a list is used, award ***(M1)*** for at least 6 correct terms seen, award ***(A1)*** for at least 8 correct terms seen.

$= 1536$ ***(A1)(G3)***

Note: Exact answer only. If both exact and rounded answer seen, award the final ***(A1)***.

[3 marks]

Examiners report

[N/A]

12d. *[3 marks]*

Markscheme

$\frac{3 \times (2^8 - 1)}{2 - 1}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted geometric series formula, ***(A1)*** for their correct substitutions. Follow through from part (c). If a list is used, award ***(M1)*** for at least 8 correct terms seen, award ***(A1)*** for these 8 correct terms being added. For consistent use of arithmetic series formula in part (d) with the arithmetic sequence formula in part (c) award a maximum of ***(M1)(A1)(A1)***.

$= 765$ ***(A1)*****(ft)*****(G2)***

[3 marks]

Examiners report

[N/A]

12e. *[3 marks]*

Markscheme

$3 \times 2^{k-1} > 600 + (k - 1)(17)$ ***(M1)***

Note: Award ***(M1)*** for their correct inequality; allow equation.
Follow through from parts (a) and (c). Accept sketches of the two functions as a valid method.

$k > 8.93648\ldots$ (may be implied) ***(A1)*****(ft)**

Note: Award ***(A1)*** for $8.93648\ldots$ seen. The GDC gives answers of -34.3 and 8.936 to the inequality; award ***(M1)(A1)*** if these are seen with working shown.

OR

$v_8 = 384\, u_8 = 719$ ***(M1)***

$v_9 = 768\, u_9 = 736$ ***(M1)***

Note: Award ***(M1)*** for v_8 and u_8 both seen, ***(M1)*** for v_9 and u_9 both seen.

$k = 9$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(G1)*** for $8.93648\ldots$ and -34.3 seen as final answer without working. Accept use of n.

[3 marks]