Logs & Exponents 2008-14 with MS

1. *[5 marks]*

Solve the equation $2 - \log_3(x+7) = \log_{\frac{1}{3}} 2x$.

2. *[5 marks]*

Let $f(x) = \ln x$. The graph of *f* is transformed into the graph of the function *g* by a translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by a reflection in the *x*-axis. Find an expression for $g(x)$, giving your answer as a single logarithm.

3. *[6 marks]*

The first terms of an arithmetic sequence are $\frac{1}{\log_2 x}, \frac{1}{\log_8 x}, \frac{1}{\log_{32} x}, \frac{1}{\log_{128} x}, \ldots$

Find *x* if the sum of the first 20 terms of the sequence is equal to 100.

4. *[5 marks]*

Solve the equation $4^{x-1} = 2^x + 8$.

5. *[6 marks]*

Solve the following system of equations.

$\log_{x+1} y = 2$

$\log_{y+1} x = \frac{1}{4}$

6. *[5 marks]*

Consider $a = \log_2 3 \times \log_3 4 \times \log_4 5 \times \ldots \times \log_{31} 32$. Given that $a \in \mathbb{Z}$, find the value of *a*.

7. *[5 marks]*

Solve the equation $8^{x-1} = 6^{3x}$. Express your answer in terms of $\ln 2$ and $\ln 3$.

Logs & Exponents 2008-14 MS

1. *[5 marks]*

Markscheme

$\log_3\left(\frac{9}{x+7}\right) = \log_3\frac{1}{2x}$ ***M1M1A1***

Note: Award ***M1*** for changing to single base, ***M1*** for incorporating the 2 into a log and ***A1*** for a correct equation with maximum one log expression each side.

$x + 7 = 18x$ ***M1***

$x = \frac{7}{17}$ ***A1***

[5 marks]

Examiners report

Some good solutions to this question and few candidates failed to earn marks on the question. Many were able to change the base of the logs, and many were able to deal with the 2, but of those who managed both, poor algebraic skills were often evident. Many students attempted to change the base into base 10, resulting in some complicated algebra, few of which managed to complete successfully.

2. *[5 marks]*

Markscheme

$h(x) = f(x-3) - 2 = \ln(x-3) - 2$ ***(M1)(A1)***

$g(x) = -h(x) = 2 - \ln(x-3)$ ***M1***

Note*:* Award **M1** only if it is clear the effect of the reflection in the *x*-axis:

the expression is correct ***OR***

there is a change of signs of the previous expression ***OR***

there's a graph or an explanation making it explicit

$= \ln e^2 - \ln(x-3)$ ***M1***

$= \ln\left(\frac{e^2}{x-3}\right)$ ***A1***

[5 marks]

Examiners report

This question was well attempted but many candidates could have scored better had they written down all the steps to obtain the final expression. In some cases, as the final expression was incorrect and the middle steps were missing, candidates scored just 1 mark. That could be a consequence of a small mistake, but the lack of working prevented them from scoring at least all method marks. Some candidates performed the transformations well but were not able to use logarithms properties to transform the answer and give it as a single logarithm.

3. *[6 marks]*

Markscheme

METHOD 1

$d = \frac{1}{\log_8 x} - \frac{1}{\log_2 x}$ ***(M1)***

$= \frac{\log_2 8}{\log_2 x} - \frac{1}{\log_2 x}$ ***(M1)***

Note: Award this ***M1*** for a correct change of base anywhere in the question.

$= \frac{2}{\log_2 x}$ ***(A1)***

$\frac{20}{2}\left(2 \times \frac{1}{\log_2 x} + 19 \times \frac{2}{\log_2 x}\right)$ ***M1***

$= \frac{400}{\log_2 x}$ ***(A1)***

$100 = \frac{400}{\log_2 x}$

$\log_2 x = 4 \Rightarrow x = 2^4 = 16$ ***A1***

METHOD 2

20^{th} term $= \frac{1}{\log_{2^{39}} x}$ ***A1***

$100 = \frac{20}{2}\left(\frac{1}{\log_2 x} + \frac{1}{\log_{2^{39}} x}\right)$ ***M1***

$100 = \frac{20}{2}\left(\frac{1}{\log_2 x} + \frac{\log_2 2^{39}}{\log_2 x}\right)$ ***M1(A1)***

Note: Award this ***M1*** for a correct change of base anywhere in the question.

$100 = \frac{400}{\log_2 x}$ ***(A1)***

$\log_2 x = 4 \Rightarrow x = 2^4 = 16$ ***A1***

METHOD 3

$\frac{1}{\log_2 x} + \frac{1}{\log_8 x} + \frac{1}{\log_{32} x} + \frac{1}{\log_{128} x} + \dots$

$\frac{1}{\log_2 x} + \frac{\log_2 8}{\log_2 x} + \frac{\log_2 32}{\log_2 x} + \frac{\log_2 128}{\log_2 x} + \dots$ ***(M1)(A1)***

Note: Award this ***M1*** for a correct change of base anywhere in the question.

$= \frac{1}{\log_2 x}(1 + 3 + 5 + \dots)$ ***A1***

$= \frac{1}{\log_2 x}\left(\frac{20}{2}(2 + 38)\right)$ ***(M1)(A1)***

$100 = \frac{400}{\log_2 x}$

$\log_2 x = 4 \Rightarrow x = 2^4 = 16$ ***A1***

[6 marks]

Examiners report

There were plenty of good answers to this question. Those who realised they needed to make each log have the same base (and a great variety of bases were chosen) managed the question successfully.

4. *[5 marks]*

Markscheme

$2^{2x-2} = 2^x + 8$ ***(M1)***

$\frac{1}{4} 2^{2x} = 2^x + 8$ ***(A1)***

$2^{2x} - 4 \times 2^x - 32 = 0$ ***A1***

$(2^x - 8)(2^x + 4) = 0$ ***(M1)***

$2^x = 8 \Rightarrow x = 3$ ***A1***

Notes: Do not award final ***A1*** if more than 1 solution is given.

[5 marks]

Examiners report

Very few candidates knew how to solve this equation. A significant number guessed the answer using trial and error after failed attempts to solve it. A number of misconceptions were identified involving properties of logarithms and exponentials.

5. *[6 marks]*

Markscheme

$\log_{x+1} y = 2$

$\log_{y+1} x = \frac{1}{4}$

so $(x+1)^2 = y$ ***A1***

$(y+1)^{\frac{1}{4}} = x$ ***A1***

EITHER

$x^4 - 1 = (x+1)^2$ ***M1***

$x = -1$, not possible ***R1***

$x = 1.70$, $y = 7.27$ ***A1A1***

OR

$1\left(x^2 + 2x + 2\right)^{\frac{1}{4}} - x = 0$ ***M1***

attempt to solve or graph of LHS ***M1***

$x = 1.70$, $y = 7.27$ ***A1A1***

[6 marks]

Examiners report

This question was well answered by a significant number of candidates. There was evidence of good understanding of logarithms. The algebra required to solve the problem did not intimidate candidates and the vast majority noticed the necessity of technology to solve the final equation. Not all candidates recognized the extraneous solution and there were situations where a rounded value of x was used to calculate the value of y leading to an incorrect solution.

6. *[5 marks]*

Markscheme

$\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \ldots \times \frac{\log 32}{\log 31}$ ***M1A1***

$= \frac{\log 32}{\log 2}$ ***A1***

$= \frac{5 \log 2}{\log 2}$ ***(M1)***

$= 5$ ***A1***

hence $a = 5$

Note: Accept the above if done in a specific base *eg* $\log_2 x$.

[5 marks]

Examiners report

[N/A]

7. *[5 marks]*

Markscheme

METHOD 1

$2^{3(x-1)} = (2 \times 3)^{3x}$ ***M1***

Note: Award ***M1*** for writing in terms of 2 and 3.

$2^{3x} \times 2^{-3} = 2^{3x} \times 3^{3x}$

$2^{-3} = 3^{3x}$ ***A1***

$\ln\left(2^{-3}\right) = \ln\left(3^{3x}\right)$ ***(M1)***

$-3\ln 2 = 3x\ln 3$ ***A1***

$x = -\frac{\ln 2}{\ln 3}$ ***A1***

METHOD 2

$\ln 8^{x-1} = \ln 6^{3x}$ ***(M1)***

$(x-1)\ln 2^3 = 3x\ln(2 \times 3)$ ***M1A1***

$3x\ln 2 - 3\ln 2 = 3x\ln 2 + 3x\ln 3$ ***A1***

$x = -\frac{\ln 2}{\ln 3}$ ***A1***

METHOD 3

$\ln 8^{x-1} = \ln 6^{3x}$ ***(M1)***

$(x-1)\ln 8 = 3x\ln 6$ ***A1***

$x = \frac{\ln 8}{\ln 8 - 3\ln 6}$ ***A1***

$x = \frac{3\ln 2}{\ln\left(\frac{2^3}{6^3}\right)}$ ***M1***

$x = -\frac{\ln 2}{\ln 3}$ ***A1***

[5 marks]

Examiners report

[N/A]