The Three Forms of a Quadratic Function (Parabola)

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Standard (or General) Form

$y = ax^2 + bx + c$

The *concavity* is determined by a.
If a > 0 the parabola is concave up.
If a < 0 the parabola is concave down.

The y-intercept is c.

The *axis of symmetry*, which is also the x-coordinate of the vertex, is $x = \frac{-b}{2a}$.

To find the x-intercepts, solve $0 = ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To get this form from the other forms:
Multiply it out & collect like terms.

Example

$y = -4x^2 + 2x - 3$

a = -4, b = 2, c = -3

The y-intercept is (0, -3).

The axis of symmetry is:

$x = \frac{-2}{(2)(-4)} = \frac{1}{4}$. So $x = \frac{1}{4}$.

Factored form

$y = a(x - \alpha)(x - \beta)$

Gives the x-intercepts: $x = \alpha, \beta$.

To find the y-intercept set $x = 0$ and evaluate.

The *concavity* is determined by a.
If a > 0 the parabola is concave up.
If a < 0 the parabola is concave down.

The x-coordinate of the vertex and the equation of the axis of the symmetry is the average of the x-intercepts, $\frac{\alpha+\beta}{2}$.

To get this form from the other forms: Factor it.

Example

$y = 2(x + 3)(x - 1)$

The zeros are $x = -3, 1$

The x-intercepts are (-3, 0), (1, 0)

The axis of symmetry is:

$x = \frac{-3+1}{2}$, so $x = -1$.

Vertex form

$y = a(x - h)^2 + k$

Gives the vertex: (h, k).
(Note the minus sign on h.)

The axis of symmetry is $x = h$.

The *concavity* is determined by a.
If a > 0 the parabola is concave up.
If a < 0 the parabola is concave down.

To get this form from the other forms: Complete the square.

To find the x-intercepts set $y = 0$ and solve for x.

To find the y-intercept set $x = 0$ and evaluate.

Example

$y = -3(x + 1)^2 - 4$

The vertex is (-1, -4)

The axis of symmetry is: $x = -1$

The Four "Quadratic Equations"

There are four ***different*** mathematical objects that are sometimes called by students the "Quadratic Equation". Try to keep them straight.

Quadratic Expression	$ax^2 + bx + c$	It can be factored, but it cannot be "solved", because it's not an equation and only equations can be solved.
Quadratic Equation	$0 = ax^2 + bx + c$	It can be solved, possibly by factoring it.
Quadratic Function	$y = ax^2 + bx + c$	It cannot be "solved." It can be graphed and the x-intercepts can be solved for, possibly by factoring it.
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	The solution of the quadratic equation.

Four statements about Quadratics

There are several ways of looking at the solutions of the quadratic equation $0 = a(x - \alpha)(x - \beta)$. The following are all the same statement:

1. The **zeros, roots** or **solutions** of a quadratic equation are α & β.
2. The **factors** of the corresponding quadratic expression are $(x - \alpha)(x - \beta)$.
3. The **zeros** of the quadratic function are α & β. These are the values of x that make $y = 0$.
4. The **x-intercepts** of the corresponding quadratic function are (α, 0), (β, 0).