Studies Quadratic Functions May & Nov 2008-2014

1a. [2 marks] A quadratic function, $f(x) = ax^2 + bx$, is represented by the mapping diagram below.



The graph cuts the y axis at (0, 4). Write down the value of q. 2b. [2 marks] The axis of symmetry is x = 2.5. Find the value of r. 2c. [1 mark] The axis of symmetry is x = 2.5. Write down the minimum value of y. 2d. [2 marks] The axis of symmetry is x = 2.5.xWrite down the range of y. 3a. [2 marks] The following is the graph of the quadratic function y = f(x). yy = f(x)/x

Write down the solutions to the equation f(x) = 0.

3b. [2 marks]

Write down the equation of the axis of symmetry of the graph of f(x).

3c. [1 mark]

The equation f(x) = 12 has two solutions. One of these solutions is x = 6. Use the symmetry of the graph to find the other solution.

3d. [1 mark]

The minimum value for *y* is – 4. Write down the range of f(x).

4a. [4 marks]

The x-coordinate of the minimum point of the quadratic function $f(x) = 2x^2 + kx + 4$ is x = 1.25.

(i) Find the value of k .

(ii) Calculate the \mathcal{Y} -coordinate of this minimum point.

4b. [2 marks]

Sketch the graph of y = f(x) for the domain $-1 \leqslant x \leqslant 3$.



5a. [2 marks]

Consider the quadratic function y = f(x), where f(x) = 5 - x + ax.

It is given that f(2) = -5. Find the value of a.

5b. [2 marks]

Find the equation of the axis of symmetry of the graph of y = f(x).

5c. [2 marks]

Write down the range of this quadratic function.

6a. [2 marks]

y = f(x) is a quadratic function. The graph of f(x) intersects the *y*-axis at the point A(0, 6) and the *x*-axis at the point B(1, 0). The vertex of the graph is at the point C(2, -2).

Write down the equation of the axis of symmetry.

6b. [3 marks]

Sketch the graph of y = f(x) on the axes below for $0 \le x \le 4$. Mark clearly on the sketch the points A, B, and C.



6c. [1 mark]

The graph of y = f(x) intersects the *x*-axis for a second time at point D. Write down the *x*-coordinate of point D.

7a. [3 marks]

Part of the graph of the quadratic function *f* is given in the diagram below.



On this graph one of the *x*-intercepts is the point (5, 0). The *x*-coordinate of the maximum point is 3. The function *f* is given by $f(x) = -x^2 + bx + c$, where $b, c \in \mathbb{Z}$ Find the value of

(i) *b*;
(ii) *c*. **7b.** [3 marks] The domain of *f* is 0 ≤ *x* ≤ 6. Find the range of *f*.

8a. [3 marks]

The graph of the quadratic function $f(x) = 3 + 4x - x^2$ intersects the y-axis at point A and has its vertex at point B .



Find the coordinates of B.

8b. [3 marks]

Another point, C, which lies on the graph of y = f(x) has the same y-coordinate as A.(i) Plot and label C on the graph above.(ii) Find the x-coordinate of C. 9a. [1 mark]

The graph of the quadratic function $f(x) = c + bx - x^2$ intersects the *y*-axis at point A(0, 5) and has its vertex at point B(2, 9).



10a. [3 marks]

The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function



The coordinates of point A are (75, 450). Determine whether point A is on the bicycle track. Give a reason for your answer.

10b. [2 marks]

Find the derivative of $y = \frac{-x^2}{10} + \frac{27}{2} x$.

10c. [4 marks]

Use the answer in part (b) to determine if A (75, 450) is the point furthest north on the track between 0 and B. Give a reason for your answer.

10d. [3 marks]

(i) Write down the midpoint of the line segment OB.

(ii) Find the gradient of the line segment OB.

10e. [3 marks]

Scott starts from a point C(0,150). He hikes along a straight road towards the bicycle track, parallel to the line segment OB.

Find the equation of Scott's road. Express your answer in the form ax + by = c, where a, b and $c \in \mathbb{R}_{-}$

10f. [2 marks]

Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track.

11a. [1 mark]

The front view of the edge of a water tank is drawn on a set of axes shown below. The edge is modelled by $y = ax^2 + c_1$.



Point P has coordinates (-3, 1.8), point O has coordinates (0, 0) and point Q has coordinates (3, 1.8).

Write down the value of *c*.

11b. [2 marks]

Find the value of *a*.

11c. [1 mark]

Hence write down the equation of the quadratic function which models the edge of the water tank. **11d.** *[2 marks]*

The water tank is shown below. It is partially filled with water.



Calculate the value of y when $x=2.4\mathrm{m}$.

11e. [2 marks]

The water tank is shown below. It is partially filled with water.

State what the value of x and the value of y represent for this water tank. **11f.** [2 marks]

The water tank is shown below. It is partially filled with water.

Find the value of x when the height of water in the tank is 0.9 m. 11g. [2 marks]

When the water tank is filled to a height of $0.9\,\text{m}$, the front cross-sectional area of the water is $2.55\,m^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is $36\ m^3$.

(ii) Calculate the percentage of water in the tank.

12a. [1 mark]

The graph of the quadratic function $f(x) = ax^2 + bx + c$ intersects the *y*-axis at point A (0, 5) and has its vertex at point B (4, 13).



Write down the value of *c*.

12b. [3 marks]

By using the coordinates of the vertex, B, or otherwise, write down **two** equations in *a* and *b*. **12c.** [2 marks]

Find the value of *a* and of *b*.

Studies Quadratic Functions May & Nov 2008-2014 MS

1a. [2 marks]

A quadratic function, $f(x) = ax^2 + bx$, is represented by the mapping diagram below.



Use the mapping diagram to write down **two** equations in terms of *a* and *b*.

Markscheme

4a + 2b = 20

a + b = 8 (A1)

a - b = -4 (A1) (C2)

Note: Award (A1)(A1) for any two of the given or equivalent equations.

[2 marks]

1b. [1 mark]

Find the value of *a*.

Markscheme

a = 2 **(A1)(ft)**

[1 mark]

1c. [1 mark]

Find the value of *b*.

Markscheme

b = 6 *(A1)*(ft) *(C2)*

Note: Follow through from their (a).

[1 mark]

1d. [2 marks]

Calculate the *x*-coordinate of the vertex of the graph of f(x).

Markscheme

$$x = -\frac{6}{2(2)} (M1)$$

Note: Award (M1) for correct substitution in correct formula.

$$= -1.5$$
 (A1)(ft) (C2)

[2 marks]

2a. [1 mark]

The graph of y = 2x - rx + q is shown for $-5 \le x \le 7$.



The graph cuts the *y* axis at (0, 4).

Write down the value of *q*.

Markscheme

q = 4 (A1) (C1)

[1 mark]

2b. [2 marks]

The axis of symmetry is x = 2.5.

Find the value of *r*.

Markscheme

$$2.5 = rac{r}{4}$$
 (M1)

r = 10 (A1) (C2)

[2 marks]

2c. [1 mark]

The axis of symmetry is x = 2.5.

Write down the minimum value of *y*.

Markscheme

-8.5 (A1)(ft) (C1)

[1 mark]

2d. [2 marks]

The axis of symmetry is x = 2.5.x

Write down the range of *y*.

Markscheme

$-8.5 \leqslant y \leqslant 104$ (A1)(ft)(A1)(ft) (C2)

Notes: Award **(***A***1)(ft)** for their answer to part (c) with correct inequality signs, **(***A***1)(ft)** for 104. Follow through from their values of *q* and *r*.

Accept 104 ±2 if read from graph.

[2 marks]

3a. [2 marks]

The following is the graph of the quadratic function y = f(x).



Write down the solutions to the equation f(x) = 0.

Markscheme

x = 0, x = 4 (A1)(A1) (C2)

Notes: Accept 0 and 4.

[2 marks]

3b. [2 marks]

Write down the equation of the axis of symmetry of the graph of f(x).

Markscheme

x = 2 (A1)(A1) (C2)

3c. [1 mark]

The equation f(x) = 12 has two solutions. One of these solutions is x = 6. Use the symmetry of the graph to find the other solution.

Markscheme

x = -2 *(A1) (C1)*

Note: Accept -2.

[1 mark]

3d. [1 mark]

The minimum value for *y* is – 4. Write down the range of f(x).

Markscheme

 $y \ge -4 (f(x) \ge -4)$ (A1) (C1)

Notes: Accept alternative notations.

Award **(A0)** for use of strict inequality.

4a. [4 marks]

The x-coordinate of the minimum point of the quadratic function $f(x)=2x^2+kx+4$ is x=1.25.

(i) Find the value of k .

(ii) Calculate the *Y*-coordinate of this minimum point.

Markscheme

(i)
$$1.25 = -\frac{k}{2(2)}$$
 (M1)

OR

$$f'(x) = 4x + k = 0$$
 (M1)

Note: Award (M1) for setting the gradient function to zero.

$$k = -5$$
 (A1) (C2)

(ii)
$$2(1.25)^2 - 5(1.25) + 4$$
 (M1)

$$= 0.875$$
 (A1)(ft) (C2)

Note: Follow through from their *k*.

[4 marks]

4b. [2 marks]

Sketch the graph of y = f(x) for the domain $-1 \leqslant x \leqslant 3$.

Markscheme





Notes: Award **(A1)(ft)** for a curve with correct concavity consistent with their *k* passing through (0, 4).

(A1)(ft) for minimum in approximately the correct place. Follow through from their part (a).

[2 marks]

5a. [2 marks]

Consider the quadratic function y = f(x), where f(x) = 5 - x + ax.

It is given that f(2) = -5. Find the value of a.

Markscheme

-5 = 5 - (2) + a(2) (M1)

Note: Award (M1) for correct substitution in equation.

(*a* =) -2 (*A1*) (*C2*)

[2 marks]

5b. [2 marks]

Find the equation of the axis of symmetry of the graph of y = f(x).

Markscheme

$$x = -\frac{1}{4}$$
 (-0.25) (A1)(A1)(ft) (C2)

Notes: Follow through from their part (a). Award (A1)(A0)(ft) for "x = constant". Award (A0)(A1)(ft) for $y = -\frac{1}{4}$.

[2 marks]

5c. [2 marks]

Write down the range of this quadratic function.

Markscheme

$f(x) \le 5.125 \ (A1)(A1)(ft) \ (C2)$

Notes: Award **(***A***1)** for $f(x) \le$ (accept y). Do not accept strict inequality. Award **(***A***1)(ft)** for 5.125 (accept 5.13). Accept other correct notation, for example, ($-\infty$, 5.125]. Follow through from their answer to part (b).

[2 marks]

6a. [2 marks]

y = f(x) is a quadratic function. The graph of f(x) intersects the *y*-axis at the point A(0, 6) and the *x*-axis at the point B(1, 0). The vertex of the graph is at the point C(2, -2).

Write down the equation of the axis of symmetry.

Markscheme

x = 2 (A1)(A1) (C2)

Notes: Award (A1)(A0) for "x = constant" (other than 2). Award (A0)(A1) for y = 2. Award (A0)(A0) for only seeing 2. Award (A0)(A0) for 2 = -b / 2a.

[2 marks]

6b. [3 marks]

Sketch the graph of y = f(x) on the axes below for $0 \le x \le 4$. Mark clearly on the sketch the points A, B, and C.



Markscheme



(A1) for correctly plotting and labelling A, B and C

(A1) for a smooth curve passing through the three given points

(A1) for completing the symmetry of the curve over the domain given. (A3) (C3)

Notes: For *A* marks to be awarded for the curve, each segment must be a reasonable attempt at a continuous curve. If straight line segments are used, penalise once only in the last two marks.

[3 marks]

6c. [1 mark]

The graph of y = f(x) intersects the *x*-axis for a second time at point D.

Write down the *x*-coordinate of point D.

Markscheme

3 (A1)(ft) (C1)

Notes: (A0) for coordinates. Accept x = 3 or D = 3.

[1 mark]

7a. [3 marks]

Part of the graph of the quadratic function *f* is given in the diagram below.



On this graph one of the *x*-intercepts is the point (5, 0). The *x*-coordinate of the maximum point is 3. The function *f* is given by $f(x) = -x^2 + bx + c$, where $b, c \in \mathbb{Z}$

Find the value of

(i) *b* ;

(ii) *c* .

Markscheme

(i)
$$3 = \frac{-b}{-2}$$
 (M1)

Note: Award (M1) for correct substitution in formula.

OR

$$-1+b+c=0$$

-25+5b+c=0

$$-24 + 4b = 0$$
 (M1)

Note: Award (M1) for setting up 2 correct simultaneous equations.

OR

-2x + b = 0 (M1)

Note: Award (M1) for correct derivative of f(x) equated to zero.

$$b = 6$$
 (A1) (C2)
(ii) $0 = -(5)^2 + 6 \times 5 + c$

$$c = -5$$
 (A1)(ft) (C1)

Note: Follow through from their value for *b*.

Note: Alternatively candidates may answer part (a) using the method below, and not as two separate parts.

$$(x-5)(-x+1)_{(M1)}$$

 $-x^2+6x-5_{(A1)}$
 $b=6\ c=-5\ (A1)\ (C3)$

[3 marks]

7b. [3 marks]

The domain of *f* is $0 \le x \le 6$.

Find the range of *f*.

Markscheme

$-5 \le y \le 4$ (A1)(ft)(A1)(ft)(A1) (C3)

Notes: Accept [-5, 4]. Award **(A1)(ft)** for -5, **(A1)(ft)** for 4. **(A1)** for inequalities in the correct direction or brackets with values in the correct order or a clear word statement of the range. Follow through from their part (a).

[3 marks]

8a. [3 marks]

The graph of the quadratic function $f(x)=3+4x-x^2$ intersects the y-axis at point A and has its vertex at point B .



Find the coordinates of B.

Markscheme

$$x = -rac{4}{-2}$$
 (M1) $x = 2$ (A1)

OR

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4 - 2x_{(M1)x} = 2_{(A1)}(2,7)_{\mathrm{or}} x = 2, y = 7_{(A1)}(C3)$$

Notes: Award (M1)(A1)(A0) for 2, 7 without parentheses.

[3 marks]

8b. [3 marks]

Another point, C , which lies on the graph of y = f(x) has the same y-coordinate as A .(i) Plot and label C on the graph above.(ii) Find the x-coordinate of C .

Markscheme

(i) C labelled in correct position on graph (A1) (C1)



(ii)
$$3 = 3 + 4x - x^2$$
 (M1)

Note: Award (M1) for correct substitution of y = 3 into quadratic.

$$(x =)4_{(A1)(C2)}$$

OR

Using symmetry of graph x = 2 + 2 . (M1)

Note: Follow through from their \boldsymbol{x} -coordinate of the vertex.

$$(x =)4$$
 (A1)(ft) (C2)

[3 marks]

9a. [1 mark]

The graph of the quadratic function $f(x) = c + bx - x^2$ intersects the *y*-axis at point A(0, 5) and has its vertex at point B(2, 9).



Write down the value of *c*.

Markscheme

5 **(A1) (C1)**

9b. [2 marks]

Find the value of *b*.

Markscheme

$$\frac{-b}{2(-1)} = 2$$
 (M1)

Note: Award **(M1)** for correct substitution in axis of symmetry formula.

OR

$$y = 5 + bx - x^2$$

$$9 = 5 + b(2) - (2)^2$$
 (M1)

Note: Award **(M1)** for correct substitution of 9 and 2 into their quadratic equation.

$$(b =)4_{(A1)(ft)(C2)}$$

Note: Follow through from part (a).

9c. [2 marks]

Find the *x*-intercepts of the graph of *f*.

Markscheme

5, -1 (A1)(ft)(A1)(ft) (C2)

Notes: Follow through from parts (a) and (b), irrespective of working shown.

9d. [1 mark]

Write down f(x) in the form f(x) = -(x-p)(x+q).

Markscheme

f(x) = -(x-5)(x+1) (A1)(ft) (C1)

Notes: Follow through from part (c).

10a. [3 marks]

The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function

$$y=rac{-x^2}{10}+rac{27}{2}\,x_{
m ,\,where}\,x\geqslant 0,\;y\geqslant 0$$

(x, y) are the coordinates of a point x metres east and y metres north of 0, where 0 is the origin (0, 0). B is a point on the bicycle track with coordinates (100, 350).



The coordinates of point A are (75, 450). Determine whether point A is on the bicycle track. Give a reason for your answer.

Markscheme

$$y = -rac{75^2}{10} + rac{27}{2} imes 75$$
 (M1)

Note: Award (M1) for substitution of 75 in the formula of the function.

Yes, point A is on the bike track. (A1)

Note: Do not award the final (A1) if correct working is not seen.

10b. [2 marks]

Find the derivative of
$$y=rac{-x^2}{10}+rac{27}{2}\,x_{.}$$

Markscheme

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x}{10} + \frac{27}{2} \left(\frac{\mathrm{d}y}{\mathrm{d}x} = -0.2x + 13.5 \right)$$
 (A1)(A1)

Notes: Award **(***A1***)** for each correct term. If extra terms are seen award at most **(***A1***)(***A0***)**. Accept equivalent forms.

10c. [4 marks]

Use the answer in part (b) to determine if A (75, 450) is the point furthest north on the track between O and B. Give a reason for your answer.

Markscheme

$$-\frac{2x}{10}+\frac{27}{2}=0$$
 (M1)

Note: Award (M1) for equating their derivative from part (b) to zero.

$$x = 67.5$$
 (A1)(ft)

Note: Follow through from their derivative from part (b).

(Their)
$$67.5 \neq 75_{(R1)}$$

Note: Award **(***R1***)** for a comparison of their 67.5 with 75. Comparison may be implied (*eg* 67.5 is the *x*-coordinate of the furthest north point).

OR

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\times(75)}{10} + \frac{27}{2}$$
 (M1)

Note: Award (M1) for substitution of 75 into their derivative from part (b).

= -1.5 (A1)(ft)

Note: Follow through from their derivative from part (b).

(Their) $-1.5 \neq 0_{(R1)}$

Note: Award *(R1)* for a comparison of their –1.5 with 0. Comparison may be implied (*eg* The gradient of the parabola at the furthest north point (vertex) is 0).

Hence A is not the furthest north point. (A1)(ft)

Note: Do not award (R0)(A1)(ft). Follow through from their derivative from part (b).

10d. [3 marks]

(i) Write down the midpoint of the line segment OB.

(ii) Find the gradient of the line segment OB.

Markscheme

(i) M(50,175) (A1)

Note: If parentheses are omitted award (A0). Accept x = 50, y = 175.

(ii)
$$\frac{350-0}{100-0}$$
 (M1)

Note: Award (M1) for correct substitution in gradient formula.

$$= 3.5 \left(\frac{350}{100}, \frac{7}{2} \right)$$
 (A1)(ft)(G2)

Note: Follow through from (d)(i) if midpoint is used to calculate gradient. Award **(G1)(G0)** for answer 3.5*x* without working.

10e. [3 marks]

Scott starts from a point C(0,150). He hikes along a straight road towards the bicycle track, parallel to the line segment OB.

Find the equation of Scott's road. Express your answer in the form ax + by = c, where a, b and $c \in \mathbb{R}$.

Markscheme

 $y = 3.5x + 150_{(A1)(ft)(A1)(ft)}$

Note: Award (A1)(ft) for using their gradient from part (d), (A1)(ft) for correct equation of line.

3.5x - y = -150 or 7x - 2y = -300 (or equivalent) (A1)(ft)

Note: Award (A1)(ft) for expressing their equation in the form ax + by = c.

10f. [2 marks]

Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track.

Markscheme

(18.4, 214) (18.3772..., 214.320...) (A1)(ft)(A1)(ft)(G2)(ft)

Notes: Follow through from their equation in (e). Coordinates must be positive for follow through marks to be awarded. If parentheses are omitted and not already penalized in (d)(i) award at most (A0)(A1)(ft). If coordinates of the two intersection points are given award (A0)(A1)(ft). Accept x = 18.4, y = 214.

11a. [1 mark]

The front view of the edge of a water tank is drawn on a set of axes shown below.



Point P has coordinates (-3, 1.8), point O has coordinates (0, 0) and point Q has coordinates (3, 1.8)

Write down the value of *c*.

Markscheme

0 (A1)(G1)

[1 mark]

11b. [2 marks]

Find the value of *a*.

Markscheme

$$1.8 = a(3)^2 + 0$$
 (M1)

OR

$$1.8 = a(-3)^2 + 0_{(M1)}$$

Note: Award (M1) for substitution of y = 1.8 or x = 3 and their value of c into equation. 0 may be implied.

$$a = 0.2 \left(\frac{1}{5}\right)$$
 (A1)(ft)(G1)

Note: Follow through from their answer to part (a).

Award **(G1)** for a correct answer only.

[2 marks]

11c. [1 mark]

Hence write down the equation of the quadratic function which models the edge of the water tank.

Markscheme

$$y = 0.2x^2$$
 (A1)(ft)

Note: Follow through from their answers to parts (a) and (b).

Answer must be an equation.

[1 mark]

11d. [2 marks]



Calculate the value of y when $x=2.4\mathrm{m}$.

Markscheme

$$0.2 \times (2.4)^2$$
 (M1)

 $= 1.15 \text{ (m)} (1.152)_{(A1)(ft)(G1)}$

Notes: Award *(M1)* for correctly substituted formula, *(A1)* for correct answer. Follow through from their answer to part (c).

Award **(G1)** for a correct answer only.

[2 marks]

11e. [2 marks]

The water tank is shown below. It is partially filled with water.



State what the value of x and the value of y represent for this water tank.

Markscheme

y is the height **(A1)**

positive value of $m{x}$ is half the width (or equivalent) (A1)

[2 marks]

11f. [2 marks]



Find the value of $m{x}$ when the height of water in the tank is 0.9 m.

Markscheme

$$0.9 = 0.2x^2$$
 (M1)

Note: Award (M1) for setting their equation equal to 0.9.

 $x = \pm 2.12 \text{ (m)} \left(\pm \frac{3}{2} \sqrt{2}, \pm \sqrt{4.5}, \pm 2.12132 \ldots \right)$ (A1)(ft)(G1)

Note: Accept 2.12. Award (G1) for a correct answer only.

[2 marks]

11g. [2 marks]



When the water tank is filled to a height of $0.9\,\text{m}$, the front cross-sectional area of the water is $2.55\,\text{m}^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is 36 m^3 .

(ii) Calculate the percentage of water in the tank.

Markscheme

(i)
$$2.55 \times 5$$
 (M1)

Note: Award (M1) for correct substitution in formula.

$$= 12.8 (m^3) (12.75 (m^3)) (A1)(G2)$$

[2 marks]

(ii)
$$\frac{12.75}{36} \times 100$$
 (M1)

Note: Award (M1) for correct quotient multiplied by 100.

= 35.4(%) (35.4166...) (A1)(ft)(G2)

Note: Award (G2) for 35.6(%)(35.5555...(%)).

Follow through from their answer to part (g)(i).

[2 marks]

12a. [1 mark]

The graph of the quadratic function $f(x) = ax^2 + bx + c$ intersects the *y*-axis at point A (0, 5) and has its vertex at point B (4, 13).



Write down the value of *c*.

Markscheme

5 **(A1) (C1)**

[1 mark]

12b. [3 marks]

By using the coordinates of the vertex, B, or otherwise, write down two equations in a and b.

Markscheme

at least one of the following equations required

$$a(4)^2 + 4b + 5 = 13$$

$$4 = -\frac{b}{2a}$$

$$a(8)^2 + 8b + 5 = 5_{(A2)(A1)}(C3)$$

Note: Award **(***A2***)(***A0***)** for one correct equation, or its equivalent, and **(***C3***)** for any two correct equations.

Follow through from part (a).

The equation $a(0)^2 + b(0) = 5$ earns no marks.

[3 marks]

12c. [2 marks]

Find the value of *a* and of *b*.

Markscheme

$a=-rac{1}{2}\,,\ b=4$ (A1)(ft)(A1)(ft) (C2)

Note: Follow through from their equations in part (b), but only if their equations lead to unique solutions for *a* and *b*.

[2 marks]

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